University of California Davis **Name (Print):** _____
Differential Equations MAT 22B **Student ID (Print):** _____

Sample Midterm Examination **October 25 2019**
Time Limit: 50 Minutes

This examination document contains 6 pages, including this cover page, and 5 problems. You must verify whether there any pages missing, in which case you should let the instructor know. **Fill in** all the requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

(A) **If you use a lemma, proposition or theorem which we have seen in the class or in the book, you must indicate this** and explain why the theorem may be applied.

(B) **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive little credit.

(C) **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive little credit; an incorrect answer supported by substantially correct calculations and explanations will receive partial credit.

(D) If you need more space, use the back of the pages; clearly indicate when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total:	100	

1. (20 points) Consider the following first-order differential equation:

$$y'(t) - 2y(t) = t.$$

(a) (10 points) Find *all* solutions to the differential equation.

(b) (10 points) Solve the Initial Value Problem given by $y(0) = 0$.

2. (20 points) Consider the first-order differential equation $y'(t) = f(y(t))$ where $f(y)$ is depicted in Figure 1, and $f(y)$ decays to minus infinity away from the picture.

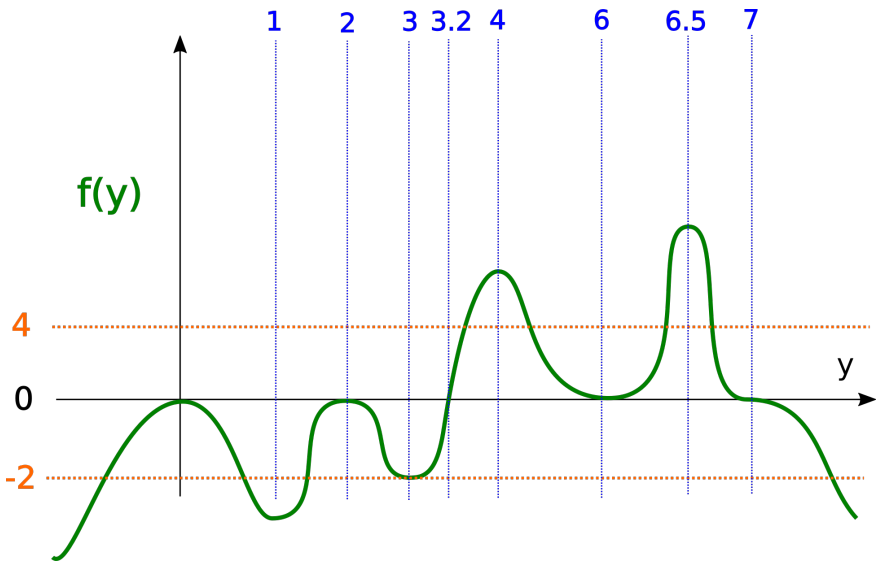

Figure 1: The function $f(y)$ for Problem 2.

(a) (5 points) How many constant solutions does $y'(t) = f(y(t))$ have ?

(b) (5 points) Classify the constant solutions into *stable*, *unstable* and *semistable*.

(c) (5 points) Describe the long term behavior of the unique solution of $y'(t) = f(y(t))$ such that $y(0) = 4$?

(d) (5 points) Find the number of constant solutions of $y'(t) = f(y(t)) + 2$.

3. (20 points) Consider the following differential equation:

$$y''(t) + 4y'(t) + 4y(t) = 3t.$$

(a) (8 points) Find a particular solution to the above differential equation.

(b) (8 points) Find *all* solutions to the above differential equation.

(c) (4 points) Is the damped harmonic oscillator described by the differential equation *overdamped*, *critically damped* or *underdamped* ?

4. (20 points) Consider the second-order differential equation:

$$t^2y''(t) - 3ty'(t) + 4y(t) = 0, \quad t > 0$$

(a) (5 points) Show that $y_1(t) = t^2$ is a solution to the differential equation above.

(b) (5 points) Suppose that $v(t)y_1(t)$ is a solution to the differential equation above. Show that $v(t)$ satisfies

$$v''(t) + t^{-1}v'(t) = 0.$$

(c) (5 points) Find a solution $y_2(t)$ which is linearly independent with $y_1(t)$.

(d) (5 points) Find all solutions to the differential equation above.

5. (20 points) For each of the ten sentences below, circle whether they are **true** or **false**.

(a) (2 points) The graphs of two solutions for $y'(t) + e^{t^9} y(t) = \cos(t^2)$ cannot intersect.

(1) True. (2) False.

(b) (2 points) The graphs of two solutions for $y''(t)+y'(t)+100y(t) = 2$ cannot intersect.

(1) True. (2) False.

(c) (2 points) Any autonomous first-order differential equation has a constant solution.

(1) True. (2) False.

(d) (2 points) The functions $y_1(t) = \sin(t)$ and $y_2(t) = \cos(t)$ have non-zero Wronskian.

(1) True. (2) False.

(e) (2 points) The local error in Euler's method with step h is of order $\mathcal{O}(h^2)$:

(1) True. (2) False.

(f) (2 points) Autonomous first-order differential equations must have finitely many constant solutions.

(1) True. (2) False.

(g) (2 points) The function $y(t) = e^{2t}$ solves the differential equation given by

$$y''(t) - 4y'(t) + 4y(t) = e^{2t}.$$

(1) True. (2) False.

(h) (2 points) The global error in Euler's method with step h is of order $\mathcal{O}(h)$.

(1) True. (2) False.

(i) (2 points) No solution $y_1(t)$ of $y'(t) = 2t$ satisfies $y_1(0) = 0$, $y_1(1) = 1$ *and* $y_1(3) = 9$.

(1) True. (2) False.

(j) (2 points) No solution $y_1(t)$ of $y'(t) = 2t$ satisfies $y_1(0) = 0$, $y_1(1) = 1$ *and* $y_1(2) = 3$.

(1) True. (2) False.