Lecture 13

Tuesday, September 29, 2020 12:05 PM

Lecture 13 : Review Session and Q&A

Proofs by contradiction : 1st step assume the statement is false. ✓

2nd step work to find a contradiction. ← try things!

Example : Show that $\exists$ infinitely many primes. ← this proof by Euclid needs to be known + Problems from PSet 2

Variations : Show that $\exists$ infinitely many primes of the form $7k+6$. ← good example problem

$4k+3$, $6k+5$

Proof by induction : State clearly which number you do induction on.

Then explain the 2 steps clearly :

Base case : State what is the base ($n=1$? $n=3$? $n=0$?), then verify it.

Induction case : Write what we assume to be true + Write what you want + how do we get what we want from what we assume?

Examples : (i) Inequalities (iii) Divisibility → also with modular arithmetic

(ii) Sums, or closed formula for series (iv) geometric problems

Recursion : (1) How to define a sequence $(x_n)_{n\in\mathbb{N}}$ by recursion

find the recursion

find some of $x_1, x_2, x_3, \dots$

come up with closed formula → geometric series $\sum_{i=1}^{n} 3^i$?

→ linear recurrence (e.g. Fibonacci)

$$X_n = 7\cdot X_{n-1} + 19\cdot X_{n-2} \quad \text{w/ } X_1 = 4,\ X_2 = 8.$$

what is X_{117}? (or a closed formula!)

charact. poly.

(2) The definition of $n!$, and $\binom{n}{k}$,

$\binom{12}{3}$ coeff.? x^3y^9 in $(x+y)^{12}$

and THE BINOMIAL THEOREM ~ see prob. of PSet 3!

$$(x+y)^n = \sum_{i=0}^{n} \binom{n}{k} \cdot x^k \cdot y^{n-k}$$

useful to solve prob.