Prob 1. Show $\exists n, m \in \mathbb{Z}$ s.t. $n^4 = 4m + 2$.

$\text{Sol}^n$: (Note that RHS is always $4m \in \mathbb{Z}$.)

By contradiction, suppose that $\exists n, m \in \mathbb{Z}$ with $n^4 = 4m + 2$.
Since $4m + 2$ is even, it must be that $n^4$ is even.
In fact, $n^4$ being even implies $n \in \mathbb{Z}$ is even.
If $n$ is even, then $n^4$ is divisible by 2. In particular, 4 | $n^4$.

Now

$n^4 - 4m = 2$, so $4 \mid \text{RHS}$, i.e., 4 | 2.

This is false, so $4 \mid \text{LHS}$.

This is a contradiction.
Problem 2: Show there are infinitely many primes.

Solution: By contradiction, assume there are finitely many primes \( \{p_1, p_2, \ldots, p_n\} \).

Consider the number \( \Omega := p_1 \cdots p_n + 1 \).

(A) First, \( \Omega \) cannot be prime. Indeed, if \( \Omega \) was prime, it'd have to be in the list, so \( \Omega = p_i \) for some \( i, 1 \leq i \leq n \). But \( \Omega \neq p_i \) by construction, as \( \Omega > p_i \).

(B) Second, \( \Omega \) is not divisible by \( p_i \). This is true as follows: if \( p_i \mid \Omega \), then note \( p_i \mid p_1 p_2 \cdots p_n \Rightarrow p_i \mid (p_1 p_2 \cdots p_n) \); but that means \( p_i \mid p_1 p_2 \cdots p_n - 1 \), so \( p_i \mid 1 \), not possible.

(A) contradicts (B), as any \( \Omega \) not prime must be divisible by a prime, so \( \exists p \mid \Omega \). \( \square \)
assume that you can
solve for a
$n \cdot 2^n$ and
Solve for $n$.

$2^n 	imes 2^k = 2^{n+k}$