Problem 7: how to do the induction step? (base case n = 1)

Suppose that k lines divide \( \mathbb{R}^2 \) into \( \frac{k^2 + k + 2}{2} \) regions.

Suppose that the (k+1)th line \( l_{k+1} \) is thrown into \( \mathbb{R}^2 \):

(i) Some regions get divided by 2, and some do not.

(ii) in brief: 

\[
\frac{k^2 + k + 2}{2} + \frac{1 + k}{2} = \frac{k^2 + k + 2 + (k+1) + 2}{2} \quad \text{check!}
\]

\[
\frac{(k+1)^2 + (k+1)}{2} \quad \text{by induction assumption.}
\]

We want

\[
\left( \frac{k^2 + k + 2}{2} \right) + (k+1) = \frac{k^2 + k + 2}{2} + (k+1)
\]

\[
\# \text{ regions w/ k lines} = \frac{k^2 + k + 2}{2}
\]

\[
\# \text{ regions w/ } (k+1) \text{-lines} = \frac{(k+1)^2 + (k+1)}{2}
\]

\[= \text{equal!} \]

Prob 3: There are infinitely many primes of the form 4k+3.

Soln: By contradiction, assume there are finitely many primes of the form 4k+3.
Call them \( \{ p_1, p_2, \ldots, p_n \} \). Consider \( P = 4(p_1 p_2 \ldots p_n) - 1 \).

(i) \( p_i \nmid P \), because if it did \( p_i \mid 1 \). Now, since
\[ P = 4k + 3, \] at least \( 3p_i \) of the form \( 4k + 3 \) dividing it.

(ii) \( p_i + P \) as \( p_i \nmid P \).

Prob. 5: (i) \[ 1 + 2 + 3 + \ldots + K = \frac{K \cdot (K + 1)}{2} \]

By induction, (i) the base case if \( K = 1 \):
\[ 1 = \frac{1 \cdot (1 + 1)}{2} = \frac{1 \cdot 2}{2} = 1 \quad \checkmark \]

(2) Induction step: we assume
\[ 1 + 2 + 3 + \ldots + K = \frac{K(K + 1)}{2} \quad \text{(k case)} \]

We want \[ 1 + 2 + 3 + \ldots + K + (K + 1) \]
\[ \approx \]
\[ \frac{(K + 1)(K + 2)}{2} \]
\[ \frac{K^2 + K + 2K + 2}{2} \]
\[ \frac{K^2 + 3K + 2}{2} \]
\[ \frac{K + K + 2}{2} \]
\[ \frac{K^2 + 2K + 2}{2} \]
\[ \checkmark \]

Prob. 6:

Base case:

Ind step:

\[ 2 \]

\[ 4 \times 4 \checkmark \]

\[ 2 \]

\[ 8 \times 8 \]

\[ K + 1 \]

\[ 3K \]
assume that you can solve for $2^x \times 2^x$ and