A problem similar to Prob 3 by induction:

Induction is on \( n \in \mathbb{N} \), and for each \( n \in \mathbb{N} \)

prove statement for all \( k \leq n \).

Example: \[ \sum_{n=0}^{n} \binom{n}{k} = 2^n \]

Solution: By induction, the base case \( n=0 \):
\[ \sum_{n=0}^{0} \binom{0}{k} = 2^0 \iff \binom{0}{0} = 2^0 \iff 1 = 1 \quad \text{TRUE} \]

Induction step: assume \( \sum_{n=0}^{n} \binom{n}{k} = 2^n \), want \( \sum_{n=0}^{n+1} \binom{n+1}{k} = 2^{n+1} \)

\[ \sum_{n=0}^{n+1} \binom{n+1}{k} = \sum_{n=0}^{n} \binom{n}{k} + \binom{n}{k} \]

\[ = 2^n + \binom{n}{k} \]

by induction

\[ \sum_{n=0}^{n+1} \binom{n+1}{k} = 2^{n+1} \]

Prob 3. (a) Once base case \( n=0 \) is done,

You assume \( \binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1} \), you want \( \binom{n+2}{k} = \binom{n+1}{k} + \binom{n+1}{k-1} \)

Prob 5. (Parenthesis) \( P_n \): # ways of correctly write \( n \) parenthesis:

\[ P_0 = 1 \]
\[ P_1 = \# \{ \text{}}} = 1 \text{, X} \]
\[ P_2 = \# \{ (()) \} = 2 \text{, (!)} \]
\[ P_3 = \# \{ (((())), ((())), ((())), ((())), ())) \} = 5 \]
\[ p_3 = \# \{ (111), (112), (121), (211), (22) \} = 5 \]

also for \( p_4, p_5, \) for \( p_4 = 14, p_5 = 42, p_6 = 132 \)

Show that:

\[ P_{n+1} = \sum_{k=0}^{n} P_k \cdot P_{n-k} \]

*Hint of Sol:*

1st method: Compute \( P_n \) directly, \( P_n = \frac{\binom{2n}{n}}{n} \)

2nd method: By recursion:

\[ p_2 = \frac{p_0 \cdot p_2 + p_1 \cdot p_1 + p_2 \cdot p_0}{2} = \frac{2+1+2}{2} = 5 \]

Think of \((a, b, c)\) or \((a, b, c, d)\)

In general, you have a product:

\[ (a_1, a_2, \ldots, a_{n-1}, a_n) \]

Depending on the \( k \) numbers left on the right, we parenthesize each side:

\[ \sum_{k=0}^{n-1} P_{n-k} \cdot P_k = P_n \]

Prob 1: 2n players, 6 players:

\[ a_n \] ways to pair-up in first round with 2n players

\[ a_1, a_2, a_3, a_4, a_5, a_6 \]
In general: suppose we have \(2n+2\) players, how many ways: \(a_{n+1}\).

Express \(a_{n+1}\) in terms of \(a_n, a_{n-1}, \ldots, a_1\).

Start at \((2n+2)\) players and pair 2 of them. How many ways can this be done?

Then we have \(2n\) players so \(a_n\) pairs:

\[ a_{n+1} = \text{?} \cdot a_n = (2n+1) \cdot a_n. \]

Counts how many pairs (2 players) we can choose out of \(2n+2\).

(1) add a different to all Piles but one

(2) parenthesize if \( \exists \) Triangle

\[(a(d(bc))) \]