**Lecture 11: Matrix Quantum Mechanics (Heisenberg 1920–1925) and After**

1. Bohr-Sommerfeld quantization conditions: 
   - For the example, \( \text{Heisenberg orbit: } R = n \cdot h \text{, } n \in \mathbb{Z} \)

2. Heisenberg's idea was to have observable 
   - \( \hat{q} = (q_0, p_0) \text{ with } \hat{E} \text{ and } \hat{q} \) different in energy

In summary, the observables \( q, p \) get quantized initially to:

\[
\hat{q}(0) = \sqrt{\frac{\hbar}{2i}} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \hat{p}(0) = \sqrt{\frac{\hbar}{2i}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}
\]

How do \( \hat{q}(0) \) and \( \hat{p}(0) \) evolve? \( \hat{q}(t), \hat{p}(t) \) are determined by Poisson bracket:
- e.g., the evolution of \( \hat{q}(t) \) depends on:

\[
\{\hat{q}(t), H\} = \frac{i}{\hbar} \{q, H\}
\]

The Hamiltonian \( H \) gets quantized:

\[
\hat{H} = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}
\]

The Hamiltonian gets diagonalized:

\[
\begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix} \begin{pmatrix} a, & b \\ c, & d \end{pmatrix} = \begin{pmatrix} E_1 a + E_2 b, & E_1 c + E_2 d \\ E_1 c + E_2 d, & E_1 a + E_2 b \end{pmatrix}
\]

- The new eigenvectors are:

\[
\begin{pmatrix} \frac{a}{\sqrt{E_1}}, & \frac{b}{\sqrt{E_2}} \\ \frac{c}{\sqrt{E_1}}, & \frac{d}{\sqrt{E_2}} \end{pmatrix}
\]

Since we cannot diagonalize \( \hat{p} \) too, the expectation \( \langle p(t) \rangle \) is different from observation.