Lecture 18: Free particle in $S^2$, hydrogen atom & other models

Monday: A problem $(R^3$ with $SO(3)$ symmetry $V(r)$ breaks into $\rightarrow \left[ \frac{-1}{2m} \Delta + V(r) \right] \psi = E \psi$

(1) study of invar for $SO(3)$

$L^2 \rightarrow SO(3)$ gives $L^2$ of dim 2

no imp revealed by $x_{cm}$ anharmon.

Lorentz symmetry $Y_{l,m}$, for $m$ fixed.

(2) 4D radial problem of hydrogen

stationary states: at least $l \geq 0$, $l = m \leq 0$ but now

more data needed because of 1D $r$-Schrodinger eq.

\[ L^2(r) - \frac{k^2}{r^2} - 2z^2 = E_m \rightarrow \text{say exist if } E = E_m = \frac{l^2}{2m}, \text{for } m = 0 \]

In conclusion, the static states are:

3 labels

\[ \psi_{n, l, m} \]

\[ Y_{n, l, m}(\theta, \phi) = Y_{l, m}(\theta, \phi) \]

\[ \psi_{n, l, m}(r) \]

\[ n, l, m \]

Hydrogen atom: the potential is $V(r) = -\frac{Z^2}{r}$

C constant depends on charge, mean, etc.

\[ \psi_{n, l, m} = Y_{l, m}(\theta, \phi) \]

\[ E = \frac{n^2}{2m}, \text{for } m = 0 \]

\[ Y_{l, m}(\theta, \phi) = \frac{1}{\sqrt{2^l l!}} \sum_{m=-l}^{l} \frac{(-1)^m}{\sqrt{(2l+1)}} \frac{\Gamma(l+m+1)}{\Gamma(l-m+1)} P_l^m(\cos \theta) e^{i m \phi} \]

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In fact $Y_{l,m}(\theta, \phi)$ is an eigenfunction of $\Delta$

\[ \Delta Y_{l,m} = -\lambda Y_{l,m} \]

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For the old model: $SU(2)$ & $SU(3)$: what are the invariants?

For $SU(3)$ the invariants are labeled by $p, q$: the space is isomorphic to $\text{dim } D(p,q) = \frac{1}{2}(p+1)(q+1)(p+q+2)$

Rank: $\text{dim } SU(2) = 3$, $\text{dim } SU(3) = 8$.

In $SU(3)$:

A group homomorphism $\rho$:

\[ \rho \in \text{dim } SU(3) \]

Example of $D(3,0)$, $p = 3$ quarks

10 dimensional $q = 0$ antiquarks