University of California Davis **Name (Print):** _____
Differential Equations MAT 22B **Student ID (Print):** _____

Sample Midterm II Examination 2 **November 20 2026**
SOLUTIONS
Time Limit: 50 Minutes

This examination document contains 13 pages, including this cover page, and 5 problems. You must verify whether there any pages missing, in which case you should let the instructor know. **Fill in** all the requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

(A) **If you use a lemma, proposition or theorem which we have seen in the class or in the book, you must indicate this** and explain why the theorem may be applied.

(B) **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive little credit.

(C) **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive little credit; an incorrect answer supported by substantially correct calculations and explanations will receive partial credit.

(D) If you need more space, use the back of the pages; clearly indicate when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total:	100	

1. (20 points) Consider the following second-order differential equation:

$$\boxed{y''(t) + 2y'(t) + 2y(t) = \lambda \cdot \delta(t - 50)}$$

depending on a real value $\lambda \in \mathbb{R}$.

(a) (5 points) Qualitatively plot the solution $y(t)$ of the differential equation such that $y(0) = 10$, $y'(0) = 0$ for each of the five values $\lambda \in \{-200, -100, 0, 100, 200\}$.

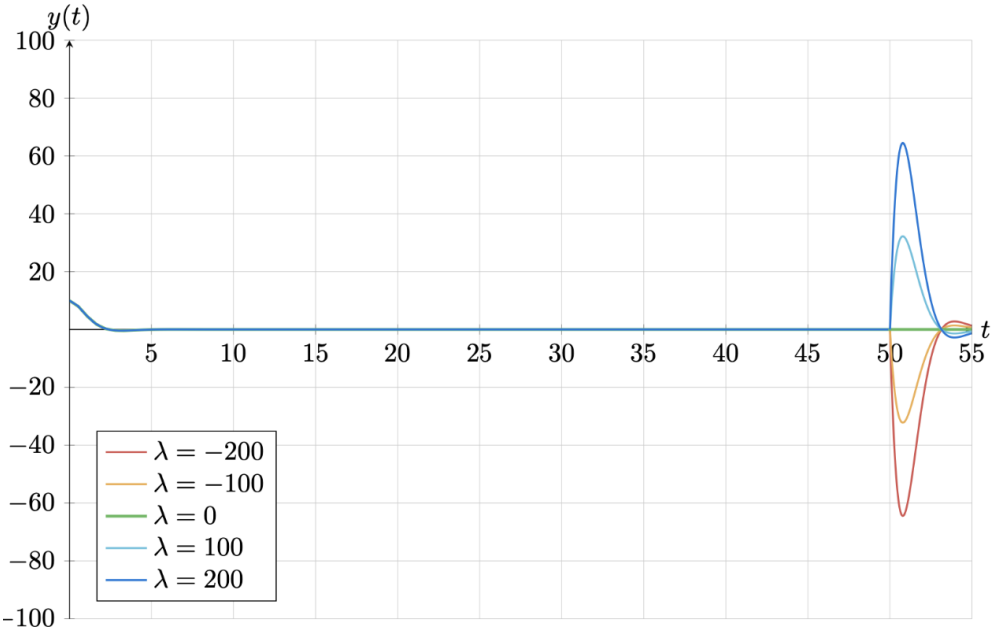
Since the forcing function on the right hand side is zero until $t = 50$, we can analyze the behavior separately for $t < 50$ and $t \geq 50$:

- For $0 \leq t < 50$: The homogeneous equation $y'' + 2y' + 2y = 0$ has roots $r = -1 \pm i$, giving a solution of the form:

$$y(t) = 10e^{-t}(\cos t + \sin t)$$

  Due to the exponential damping factor e^{-t}, by $t \approx 50$, $y(t)$ and $y'(t)$ have decayed very close to zero.
- At $t = 50$: The impulse $\lambda\delta(t-50)$ creates a sudden jump in velocity: $\Delta y(50) = 0$ and $\Delta y'(50) = \lambda$.
- For $t \geq 50$: We should expect the system to return to damped oscillation (note that it is reasonable to solve part (b) first to inform your plot).

Given this information, we can qualitatively plot solutions:



(b) (10 points) Find *all* solutions to the differential equation for $\lambda = 100$.

Taking the Laplace transform of $y'' + 2y' + 2y = 100\delta(t-50)$ and using the notation $F(s)$ for the Laplace transform of y, we get:

$$(s^2F(s) - sy(0) - y'(0)) + 2(sF(s) - y(0)) + 2F(s) = 100e^{-50s}$$

Plug in $y(0) = 10, y'(0) = 0$:

$$(s^2 + 2s + 2)F(s) - 10s - 20 = 100e^{-50s}$$

$$(s^2 + 2s + 2)F(s) = 10(s+2) + 100e^{-50s}$$

$$F(s) = \frac{10(s+2)}{s^2+2s+2} + 100e^{-50s}\frac{1}{s^2+2s+2}$$

Rewrite the first component so that we can recognize it as the Laplace transform of functions we know:

$$\frac{10(s+1)+10}{s^2+2s+2} = \frac{10(s+1)+10}{(s+1)^2+1} = 10\frac{s+1}{(s+1)^2+1} + 10\frac{1}{(s+1)^2+1}$$

Taking the inverse Laplace transform $\mathcal{L}^{-1}$:

$$y(t) = 10e^{-t}\cos t + 10e^{-t}\sin t + 100u_{50}(t)e^{-(t-50)}\sin(t-50)$$

(c) (5 points) Does the long term behavior $\lim_{t\to\infty} y(t)$ of the solutions with $y(0) = 10$, $y'(0) = 0$ of the original differential equation depend on λ? (Justify your answer.)

Answer: NO. This is because every term in the general solution has a damping factor of e^{-t}, and is multiplied by a sine, cosine, or step function. Therefore e^{-t} dominates every term in the limit regardless of the value of λ, and every solution tends towards 0.

2. (20 points) Consider the following initial value problem:

$$\boxed{y^{(4)}(t) + 5y''(t) + 4y(t) = u_1(t) - u_3(t) + \delta(t-5)}$$

$$y(0) = y'(0) = y''(0) = y'''(0) = 0.$$

(a) (10 points) Justify that the following two Laplace transforms are correct:

$$\mathcal{L}\left(\frac{1}{4} - \frac{1}{3}\cos t + \frac{1}{12}\cos(2t)\right) = \frac{1}{s(s^2+1)(s^2+4)}$$

$$\mathcal{L}\left(\frac{1}{3}\sin t - \frac{1}{6}\sin(2t)\right) = \frac{1}{(s^2+1)(s^2+4)}$$

- To show $\mathcal{L}\left(\frac{1}{4} - \frac{1}{3}\cos t + \frac{1}{12}\cos(2t)\right) = \frac{1}{s(s^2+1)(s^2+4)}$:

$$\mathcal{L}\left(\frac{1}{4} - \frac{1}{3}\cos t + \frac{1}{12}\cos(2t)\right) = \frac{1}{4s} - \frac{s}{3(s^2+1)} + \frac{s}{12(s^2+4)}$$

Combining over a common denominator $12s(s^2+1)(s^2+4)$:

$$= \frac{3(s^2+1)(s^2+4) - 4s^2(s^2+4) + s^2(s^2+1)}{12s(s^2+1)(s^2+4)}$$

$$= \frac{3(s^4+5s^2+4) - 4s^4 - 16s^2 + s^4 + s^2}{12s(s^2+1)(s^2+4)}$$

$$= \frac{(3-4+1)s^4 + (15-16+1)s^2 + 12}{12s(s^2+1)(s^2+4)} = \frac{12}{12s(s^2+1)(s^2+4)} = \frac{1}{s(s^2+1)(s^2+4)}$$

- To show $\mathcal{L}\left(\frac{1}{3}\sin t - \frac{1}{6}\sin(2t)\right) = \frac{1}{(s^2+1)(s^2+4)}$:

$$\mathcal{L}\left(\frac{1}{3}\sin t - \frac{1}{6}\sin(2t)\right) = \frac{1}{3(s^2+1)} - \frac{2}{6(s^2+4)} = \frac{1}{3}\left[\frac{1}{s^2+1} - \frac{1}{s^2+4}\right]$$

$$= \frac{1}{3}\cdot\frac{(s^2+4)-(s^2+1)}{(s^2+1)(s^2+4)} = \frac{1}{3}\cdot\frac{3}{(s^2+1)(s^2+4)} = \frac{1}{(s^2+1)(s^2+4)}$$

(b) (10 points) Show that the function

$$y(t) = u_1(t)f(t-1) - u_3(t)f(t-3) + u_5(t)g(t-5).$$

is the unique solution to the initial value problem, where we denoted

$$f(t) := \frac{1}{4} - \frac{1}{3}\cos t + \frac{1}{12}\cos(2t),$$

$$g(t) := \frac{1}{3}\sin t - \frac{1}{6}\sin(2t).$$

Taking the Laplace transform of the differential equation with all zero initial conditions:

$$(s^4 + 5s^2 + 4)Y(s) = \frac{e^{-s}}{s} - \frac{e^{-3s}}{s} + e^{-5s}$$

Since $s^4 + 5s^2 + 4 = (s^2 + 1)(s^2 + 4)$:

$$Y(s) = e^{-s}\frac{1}{s(s^2 + 1)(s^2 + 4)} - e^{-3s}\frac{1}{s(s^2 + 1)(s^2 + 4)} + e^{-5s}\frac{1}{(s^2 + 1)(s^2 + 4)}$$

Let $F(s) = \mathcal{L}(f(t)) = \frac{1}{s(s^2+1)(s^2+4)}$ and $G(s) = \mathcal{L}(g(t)) = \frac{1}{(s^2+1)(s^2+4)}$.

$$Y(s) = e^{-s}F(s) - e^{-3s}F(s) + e^{-5s}G(s)$$

Applying the time-shift theorem $\mathcal{L}^{-1}(e^{-cs}H(s)) = u_c(t)h(t - c)$:

$$y(t) = u_1(t)f(t - 1) - u_3(t)f(t - 3) + u_5(t)g(t - 5)$$

By existence and uniqueness theorems for linear IVPs, this is the unique solution.

3. (20 points) Consider the following linear system of differential equations:

$$\begin{pmatrix} x_1'(t) \\ x_2'(t) \\ x_3'(t) \end{pmatrix} = \begin{pmatrix} 3 & 1 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix} + \begin{pmatrix} e^t \\ t \\ 0 \end{pmatrix}$$

(a) (10 points) Find two linearly independent solutions to **homogeneous** system.

The homogeneous matrix $A = \begin{pmatrix} 3 & 1 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ has eigenvalue $\lambda = 3$ with multiplicity 3. Eigenvector equation:

$$(A - 3I)\vec{v} = \mathbf{0} \implies \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \implies v_2 = 0$$

We pick two linearly independent eigenvectors:

$$\vec{v}^{(1)} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \vec{v}^{(3)} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Thus, two linearly independent homogeneous solutions are:

$$\vec{x}^{(1)}(t) = e^{3t} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \vec{x}^{(3)}(t) = e^{3t} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

(b) (5 points) Find all solutions to the **homogeneous** system.

To construct the complete basis, we find a generalized eigenvector $\vec{w}$ solving $(A - 3I)\vec{w} = \vec{v}^{(1)}$:

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \implies w_2 = 1$$

Choosing $w_1 = w_3 = 0$ gives $\vec{w} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$. The second solution is:

$$\vec{z}(t) = e^{3t}(t\vec{v}^{(1)} + \vec{w}) = e^{3t} \begin{pmatrix} t \\ 1 \\ 0 \end{pmatrix}$$

This gives the general homogeneous solution:

$$\vec{x}_h(t) = c_1 e^{3t} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + c_2 e^{3t} \begin{pmatrix} t \\ 1 \\ 0 \end{pmatrix} + c_3 e^{3t} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

(c) (5 points) Find all solutions to the original **non-homogeneous** system.

We can use undetermined coefficients to find a particular solution (now using vectors instead of constants). Since there is an e^t term and a polynomial of degree 1 (t) in the forcing vector, we will assume that $\vec{x}(t) = \vec{a}e^t + \vec{b}t + \vec{c}$ where each vector has three components (i.e. $\vec{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$).

Taking a derivative, we see that $\vec{x}'(t) = \vec{a}e^t + \vec{b}$. Plugging things into the differential equation gives the following equations:

$$\begin{aligned} x_1'(t) &= 3x_1(t) + x_2(t) + e^t \\ x_2'(t) &= 3x_2(t) + t \\ x_3'(t) &= 3x_3(t) \end{aligned}$$

(Aside: note that at this point I haven't guessed a solution form for $\vec{x}(t)$ yet, and because the coefficient matrix is mostly diagonal I could just solve the system by treating the second two equations as single ODEs and substituting the solution for x_2 into the first equation. This involves using the integrating factor method a few times)

Now we plug in our guess for $\vec{x}(t) = \vec{a}e^t + \vec{b}t + \vec{c}$ to get the following system of equations:

$$\begin{aligned} a_1 e^t + b_1 &= 3(a_1 e^t + b_1 t + c_1) + a_2 e^t + b_2 t + c_2 + e^t \\ a_2 e^t + b_2 &= 3(a_2 e^t + b_2 t + c_2) + t \\ a_3 e^t + b_3 &= 3(a_3 e^t + b_3 t + c_3) \end{aligned}$$

For each equation, we can compare the coefficients of e^t, of t, and the constants to get the following 9 equations:

$$\begin{aligned} a_1 &= 3a_1 + a_2 + 1 \\ 0 &= 3b_1 + b_2 \\ b_1 &= 3c_1 + c_2 \\ a_2 &= 3a_2 \\ 0 &= 3b_2 + 1 \\ b_2 &= 3c_2 \\ a_3 &= 3a_3 \\ 0 &= 3b_3 \\ b_3 &= 3c_3 \end{aligned}$$

Solving this system of equations gives the following values:

$$a_3 = b_3 = c_3 = 0, a_2 = 0, b_2 = -1/3, c_2 = -1/9, a_1 = -1/2, b_1 = 1/9, c_1 = 2/27$$

Therefore the particular solution is

$$\vec{x}_p(t) = \begin{pmatrix} -\frac{1}{2}e^t + \frac{1}{9}t + \frac{2}{27} \\ -\frac{1}{3}t - \frac{1}{9} \\ 0 \end{pmatrix}$$

and adding the particular solution to the homogeneous solution gives a final answer of

$$\vec{x}(t) = c_1 e^{3t} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + c_2 e^{3t} \begin{pmatrix} t \\ 1 \\ 0 \end{pmatrix} + c_3 e^{3t} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + \begin{pmatrix} -\frac{1}{2}e^t + \frac{1}{9}t + \frac{2}{27} \\ -\frac{1}{3}t - \frac{1}{9} \\ 0 \end{pmatrix}$$

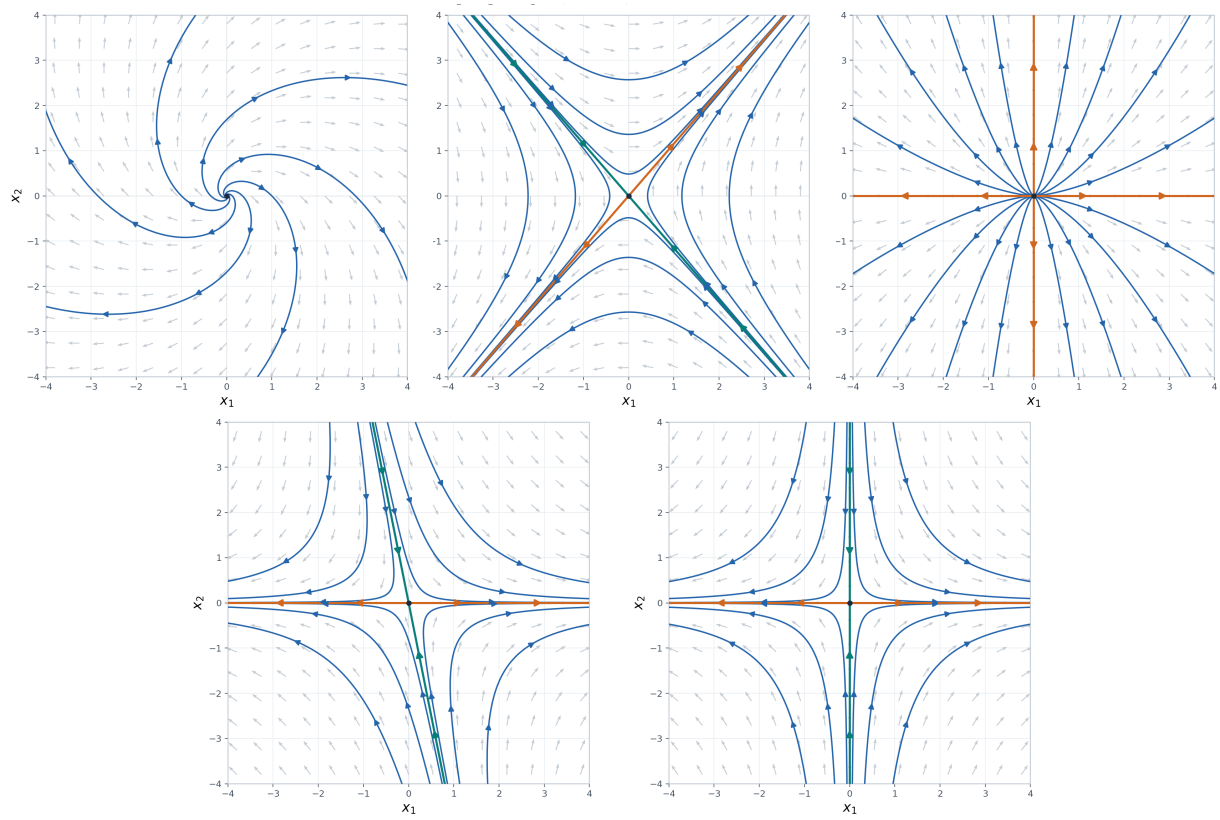


Figure 1: Phase portraits from Problem 4

4. (20 points) Consider a 2×2 system of differential equations $\dot{\vec{x}}(t) = A\vec{x}$ and the following 5 matrices A:

$$\begin{pmatrix} 2 & 0 \\ 0 & -3 \end{pmatrix}, \quad \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}, \quad \begin{pmatrix} 2 & 1 \\ 0 & -3 \end{pmatrix}, \quad \begin{pmatrix} 0 & 3 \\ 4 & 0 \end{pmatrix}, \quad \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}.$$

(a) (10 points) Match each of the phase portraits in Figure 1 with the corresponding system $\dot{\vec{x}}(t) = A\vec{x}$ for each of the 5 matrices above. (Justify your answers.)

- $A_1 = \begin{pmatrix} 2 & 0 \\ 0 & -3 \end{pmatrix}$: Eigenvalues $\lambda = 2, -3$. Saddle point aligned with coordinate axes (x_1-axis unstable, x_2-axis stable). $\longrightarrow$ **Bottom Right**.
- $A_2 = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$: Eigenvalues $\lambda = 2, 3$. Unstable node (star source) with straight radial lines. $\longrightarrow$ **Top Right**.
- $A_3 = \begin{pmatrix} 2 & 1 \\ 0 & -3 \end{pmatrix}$: Eigenvalues $\lambda = 2, -3$. Saddle point with unstable eigenvector $(1, 0)^T$ and tilted stable eigenvector $(1, -5)^T$. $\longrightarrow$ **Bottom Left**.

- $A_4 = \begin{pmatrix} 0 & 3 \\ 4 & 0 \end{pmatrix}$: Eigenvalues $\lambda = \pm\sqrt{12} = \pm 2\sqrt{3}$. Saddle point with diagonal asymptotic lines $x_2 = \pm\frac{2}{\sqrt{3}}x_1$. $\longrightarrow$ **Top Middle**.
- $A_5 = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$: Complex eigenvalues $\lambda = 1 \pm i$. Unstable spiral source rotating clockwise. $\longrightarrow$ **Top Left**.

(b) (5 points) Suppose that we add the matrix

$$E_\varepsilon := \begin{pmatrix} 0 & \varepsilon \\ 0 & 0 \end{pmatrix}$$

to each of the matrices A above, with $\varepsilon \in \mathbb{R}_{>0}$ arbitrary small. Explain how the phase portraits of $\dot{\vec{x}}(t) = (A + E_\varepsilon)\vec{x}$ qualitatively change with respect to those of $\dot{\vec{x}}(t) = A\vec{x}$. (Justify your answer.)

Because the real parts of all eigenvalues for A_1, A_2, A_3, A_4, A_5 are strictly non-zero, adding E_ϵ for small $\epsilon > 0$ alters eigenvector slopes slightly but preserves all qualitative features (saddles remain saddles, nodes remain nodes, and spiral sources remain spiral sources).

(c) (3 points) Consider the following system of differential equations

$$\begin{pmatrix} x_1'(t) \\ x_2'(t) \end{pmatrix} = \begin{pmatrix} 0 & 3 \\ 4 & 0 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}, \qquad \begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$$

Describe the long term behavior of $x_1(t)$ and $x_2(t)$.

Eigenvalues are $\lambda_1 = 2\sqrt{3} > 0$ and $\lambda_2 = -2\sqrt{3} < 0$ with corresponding eigenvectors $\vec{v}_1 = \begin{pmatrix} \sqrt{3} \\ 2 \end{pmatrix}$ and $\vec{v}_2 = \begin{pmatrix} \sqrt{3} \\ -2 \end{pmatrix}$. Decomposing $\vec{x}(0) = \begin{pmatrix} 5 \\ 1 \end{pmatrix} = c_1\vec{v}_1 + c_2\vec{v}_2$ gives:

$$c_1 = \frac{5 + \sqrt{3}/2}{2\sqrt{3}} > 0$$

As $t \to \infty$, the term $c_1 e^{2\sqrt{3}t}\vec{v}_1$ dominates completely. Therefore, both $x_1(t) \to +\infty$ and $x_2(t) \to +\infty$, with the trajectory aligning asymptotically along the ray $x_2 = \frac{2}{\sqrt{3}}x_1$ in Quadrant I.

(d) (2 points) Consider the following system of differential equations

$$\begin{pmatrix} x_1'(t) \\ x_2'(t) \end{pmatrix} = \begin{pmatrix} 0 & 3 \\ \lambda & 0 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}, \qquad \begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$$

Describe the phase portraits for the different values of $\lambda \in [0, 4]$.

The matrix $\begin{pmatrix} 0 & 3 \\ \lambda & 0 \end{pmatrix}$ has eigenvalues $r = \pm\sqrt{3\lambda}$.

- $\lambda = 0$**:** Repeated eigenvalue $r = 0$. System becomes $x_1' = 3x_2, x_2' = 0$. Trajectories are horizontal lines moving rightward for $x_2 > 0$ and leftward for $x_2 < 0$.
- $0 < \lambda \leq 4$**:** Real eigenvalues of opposite signs $r = \pm\sqrt{3\lambda}$. The origin is a saddle point; as λ increases toward 4, the speed of motion along trajectories increases and the stable/unstable eigenvectors tilt toward $x_2 = \pm\sqrt{\frac{\lambda}{3}}x_1$.

5. (20 points) For each of the five sentences below, circle the unique correct answer.

(a) (4 points) $c_1 e^{2t}\begin{pmatrix}1\\0\end{pmatrix} + c_2 e^{2t} v_2$ solves $\begin{pmatrix}x_1'(t)\\x_2'(t)\end{pmatrix} = \begin{pmatrix}2&1\\0&2\end{pmatrix}\begin{pmatrix}x_1(t)\\x_2(t)\end{pmatrix}$ if v_2 is

(1) $\begin{pmatrix}1\\0\end{pmatrix}$ (2) $\begin{pmatrix}0\\1\end{pmatrix}$ (3) $\begin{pmatrix}1\\t\end{pmatrix}$ (4) $\begin{pmatrix}t\\1\end{pmatrix}$

Quick justification: we can use a generalized eigenvector z here for a repeated eigenvalue 2 and the eigenvector $v_1 = \begin{pmatrix}1\\0\end{pmatrix}$. The formula is $z = tv_1 + w$ where w is given by $(A - \lambda I)w = v$

(b) (4 points) If $A = \begin{pmatrix}-4&1\\0&-4\end{pmatrix}$, the exponential e^{tA} is

(1) $e^{-4t}\begin{pmatrix}1&1\\0&1\end{pmatrix}$ (2) $e^{-4t}\begin{pmatrix}1&0\\0&1\end{pmatrix}$ (3) $e^{-4t}\begin{pmatrix}1&t\\0&1\end{pmatrix}$ (4) $e^{-4t}\begin{pmatrix}1&1+t\\0&1\end{pmatrix}$

Quick justification: Decompose $A = -4I + N$ where $N = \begin{pmatrix}0&1\\0&0\end{pmatrix}$ is nilpotent ($N^2 = 0$). We will use the Taylor series expansion of e^x to define e^{tA}. Since $-4I$ and N commute:

$$e^{tA} = e^{-4tI}e^{tN} = (e^{-4t}I)(I + tN) = e^{-4t}\begin{pmatrix}1&t\\0&1\end{pmatrix}$$

(c) (4 points) A particular solution to $\begin{pmatrix}x_1'(t)\\x_2'(t)\end{pmatrix} = \begin{pmatrix}2&1\\0&2\end{pmatrix}\begin{pmatrix}x_1(t)\\x_2(t)\end{pmatrix} + \begin{pmatrix}t\\e^t\end{pmatrix}$ is

(1) $\begin{pmatrix}e^{-t} - t/2 - 1/4\\-e^{-t}\end{pmatrix}$ (2) $\begin{pmatrix}e^t - t^2/2 - 1/4\\-e^t\end{pmatrix}$ (3) $\begin{pmatrix}e^t - t/2 - 1/4\\-e^t\end{pmatrix}$

Quick justification: we can either just compute and check that (1) and (2) do not satisfy the system of equations, or we can use undetermined coefficients to find (3) as a particular solution.

(d) (4 points) Consider the solution $y(t)$ of $y''(t) + 16y(t) = \delta(t-1) + \delta(t-c)$, with $y(0) = y'(0) = 0$ and $c > 1$. Then, for all time $t > c$, $y(t)$ is

(1) Never zero for any $c > 1$. (2) Always zero for all $c > 1$.

(3) Zero only for finitely many $c > 1$. (4) Zero for infinitely many $c > 1$.

Quick justification: taking the Laplace transform and solving for $y(t)$ gives sines and cosines; the oscillating behavior has infinitely many zeroes.

(e) (4 points) The Laplace transform of $e^{3t}\cos(4t)$ is:

(1) $\frac{s}{s^2+16}$ (2) $\frac{s}{(s-3)^2+16}$ (3) $\frac{s-3}{(s-3)^2+16}$ (4) $\frac{s-3}{s^2+16}$ (4) $\frac{s+3}{s^2+16}$

Quick justification: Since $\mathcal{L}(\cos(4t)) = \frac{s}{s^2+16}$, using the shift $\mathcal{L}(e^{at}f(t)) = F(s-a)$ with $a = 3$:

$$\mathcal{L}(e^{3t}\cos(4t)) = \frac{s-3}{(s-3)^2+16}$$