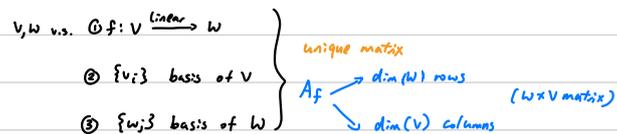


Linear maps to matrices



Example:

$f: \mathbb{R}^3 \xrightarrow{\text{linear}} \mathbb{R}^2$

$\{v_i\} = \{(1,0,0), (0,1), (1,2)\}$

$\{w_j\} = \{(3,0), (2,1)\}$

$A_f = \begin{pmatrix} * & * & * \\ * & * & * \end{pmatrix}$

2x3 matrix

Def: Linear maps to matrix

Let V, W be v.s., $f: V \rightarrow W$ & $\{v_i\}, \{w_j\}$ basis of V, W , The matrix A_f associated to f in the basis $\{v_i\}, \{w_j\}$

$A_f = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & \dots & \dots & \vdots \\ \vdots & \dots & \dots & \vdots \\ a_{m1} & \dots & \dots & a_{mn} \end{pmatrix}$ $m \times n$ matrix

where $f(v_i) = a_{1i}w_1 + \dots + a_{mi}w_m$

$\exists! a_{ji} \in \mathbb{R}$

Example

$f: \mathbb{R}^3 \xrightarrow{\text{linear}} \mathbb{R}^2$

$\{v_i\} = \{(1,0,0), (0,1), (1,2)\}$

$\{w_j\} = \{(3,0), (2,1)\}$

choose $f(x_1, x_2, x_3) = (x_1 - 3x_2, x_2 + x_3)$

$v_1 \mapsto f(v_1) = (1,0) = a_{11}(3,0) + a_{21}(2,1)$

$= \frac{1}{3}(3,0) + 0(2,1)$

$v_2 \mapsto f(v_2) = (-3,2) = a_{12}(3,0) + a_{22}(2,1)$

$= -\frac{3}{3}(3,0) + 2(2,1)$

$v_3 \mapsto f(v_3) = (-2,3) = a_{13}(3,0) + a_{23}(2,1)$

$= -\frac{2}{3}(3,0) + 3(2,1)$

$A_f = \begin{pmatrix} \frac{1}{3} & -\frac{3}{3} & -\frac{2}{3} \\ 0 & 2 & 3 \end{pmatrix}$

multiplication with $v \in V$ yields coordinates in Basis coordinates

Ex. 2. Different bases for same map give different matrices!

$V = W = \mathbb{R}^3, \{v_i\} = \{w_j\}$ for this example

First choice: $\{v_i\} = \{(1,0,0), (0,1,0), (0,0,1)\}$

$\{w_j\} = \{(0,0,0), (0,1,0), (0,0,1)\}$

$A_f = 3 \times 3$ matrix ($f: \mathbb{R}^3 \rightarrow \mathbb{R}^3$)

$f(x_1, x_2, x_3) = (x_1 + 2x_2 + x_3, 6x_1 - x_2, -x_1 - 2x_2 - x_3)$

$A_f = \begin{pmatrix} 1 & 2 & 1 \\ 6 & -1 & 0 \\ -1 & -2 & -1 \end{pmatrix}$

Second choice: $\{v_i\} = \{(-1, -6, 13), (-3, -3, 2), (-1, 2, 1)\} = \{w_j\}$

$$v_1 \mapsto f(v_1) = (0, 0, 0) \leftarrow a_{i1} = 0$$

$$v_2 \mapsto f(v_2) = (-6, -9, 6) = 0 \cdot w_1 + 3w_2 + 0w_3$$

$$v_3 \mapsto f(v_3) = (4, -8, -4) = 0 \cdot w_1 + 0 \cdot w_2 - 4w_3$$

$$A_f = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -4 \end{pmatrix}$$