# LECTURE 13: PRACTICE EXERCISES

MAT-67 SPRING 2024

Abstract. These practice problems correspond to the 13th lecture of MAT-67 Spring 2024, delivered on April 29th 2024.

The following are practice problems. They are not to be submitted, they are for your own practice. Solutions will be posted soon.

**Problem 1**. Solve the following parts:

(1) Let $V = W = \mathbb{R}^2$ and consider the basis $\{v_1, v_2\}$ and $\{w_1, w_2\}$ given by
$$v_1 = (1, 0), \quad v_2 = (0, 1),$$
$$w_1 = (2, -1), \quad w_2 = (3, 4).$$
Find a formula for the unique linear map $f : V \longrightarrow W$ such that $f(v_i) = w_i$.

(2) Let $V = W = \mathbb{R}^3$ and consider the basis $\{v_1, v_2, v_3\}$ and $\{w_1, w_2, w_3\}$ given by
$$v_1 = (1, 0, 0), \quad v_2 = (0, 1, 0), \quad v_3 = (1, 1, 1)$$
$$w_1 = (2, -1, 0), \quad w_2 = (3, 4, 0), \quad w_3 = (1, 2, 1).$$
Find a formula for the unique linear map $f : V \longrightarrow W$ such that $f(v_i) = w_i$.

**Problem 2**. Suppose that a linear map $f : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ satisfies
$$f(0, 0, 1) = (1, 3, 2), \quad f(0, 1, 0) = (0, 0, -6), \quad f(0, 0, 1) = (2, -3, 1).$$

(1) Find $f(4, 3, -6)$ and $f(5, 1, 2)$.

(2) Show that $f(v) = 0$ implies $v = 0$.

(3) Is there a vector $w \in \mathbb{R}^3$ such that there is no $v \in \mathbb{R}^3$ with $f(v) = w$?

**Problem 3**. Let $V = W = \mathbb{R}[x]$ and $f : V \longrightarrow W$ the unique linear map such that $f(x^n) = nx^{n-1}$.

(1) Find $f(1 - 3x + 5x^2 - x^7)$.

(2) Find $f^3(1 - 3x + 5x^2 - x^7)$, where $f^3 = f \circ f \circ f$ is the composition of $f$ with itself 3 times.

**Problem 4**. Let $V = \mathbb{R}^4$. Solve the following parts.

(1) Give an example of two linear maps $f, g : V \longrightarrow V$ such that $f \circ g \neq g \circ f$.

(2) Suppose that $f(1, 1, 3, 1) = (0, 0, 0, 0)$, $f(0, 2, 6, 0) = (0, 0, 0, 0)$ and $f(0, 0, 0, 7) = (0, 0, 0, 0)$. Show that $f(1, 0, 0, 0) = (0, 0, 0, 0)$.

(3) Give an example of a linear map $f : V \longrightarrow V$ such that $f \circ f = \mathrm{Id}$ is the identity.