LECTURE 4: PRACTICE EXERCISES

MAT-67 SPRING 2024

Abstract. These practice problems correspond to the 4th lecture of MAT-67 Spring 2024, delivered on April 8th 2024.

The following are practice problems. They are not to be submitted, they are for your own practice. Solutions will be posted soon.

Problem 1. Prove or disprove whether each of the following sets V if a vector space. If sum and scalar multiplication are not specified, they are just taken to be the usual sum in $\mathbb{R}^n$ or of polynomials.

(1) The subset $V \subseteq \mathbb{R}^2$ of all vectors of the form
$$V = \{(x_1, x_2) \in \mathbb{R}^2 : x_1 + x_2 = 0\}.$$
(2) The subset $V \subseteq \mathbb{R}^2$ of all vectors of the form
$$V = \{(x_1, x_2) \in \mathbb{R}^2 : x_1 + x_2 = 1\}.$$
(3) The subset $V \subseteq \mathbb{R}^2$ of all vectors of the form
$$V = \{(x_1, x_2) \in \mathbb{R}^2 : x_1 x_2 = 1\}.$$
(4) The subset $V \subseteq \mathbb{R}^4$ of all vectors of the form
$$V = \{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 : x_1 = 0\}.$$
(5) The subset $V \subseteq \mathbb{R}^4$ of all vectors of the form
$$V = \{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 : x_1 = -2\}.$$
(6) The subset $V \subseteq \mathbb{R}^2$ of all vectors of the form
$$V = \{(x_1, x_2) \in \mathbb{R}^2 : x_1 x_2 = 0\}.$$
(7) The solutions $(x_1, x_2, x_3) \in \mathbb{R}^3$ of the system of equations:
$$\begin{cases} x_1 + x_2 + x_3 = 0 \\ 2x_1 + 2x_2 + 6x_3 = 0 \\ x_1 + x_2 + 5x_3 = 0 \end{cases}$$
(8) The solutions $(x_1, x_2, x_3) \in \mathbb{R}^3$ of the system of equations:
$$\begin{cases} x_1 + x_2 + x_3 = 2 \\ 2x_1 + 2x_2 + 6x_3 = -3 \\ x_1 + x_2 + 5x_3 = 0 \end{cases}$$

(9) The solutions $(x_1, x_2, x_3) \in \mathbb{R}^3$ of the system of equations:

$$\begin{cases} x_1 + x_2 x_3 = 0 \\ 2x_1 + 2x_2 + 6x_3 = 0 \\ x_1 + x_2 + 5x_3 = -1 \end{cases}$$

(10) The subspace $V \subseteq \mathbb{R}[x]$ of polynomials of the form

$$V = \{p(x) \in \mathbb{R}[x] : p(3) = 0\}$$

(11) The subspace $V \subseteq \mathbb{R}[x]$ of polynomials of the form

$$V = \{p(x) \in \mathbb{R}[x] : p(0) = 2\}$$

(12) The subspace $V \subseteq \mathbb{R}[x]$ of polynomials of the form

$$V = \{p(x) \in \mathbb{R}[x] : p(1) = 0, p(2) = 0, p(4.5) = 0\}$$

Problem 2. Let V be an $\mathbb{R}$-vector space. Solve the following parts:

(1) In every vector space the additive identity is unique.

(2) Every vector $v \in V$ has a unique additive inverse.

(3) Scalar multiplication by $0 \in \mathbb{R}$ must always be the zero vector $\vec{0} \in V$, i.e. prove

$$0 \cdot \vec{v} = \vec{0}, \quad \forall \vec{v} \in V$$

(4) Show that for any scalar $a \in \mathbb{R}$, $a \cdot \vec{0} = \vec{0}$.

(5) Show that $(-1) \cdot \vec{v} = -\vec{v}$ for any vector $v \in V$.

Hint: think carefully about what the left hand side and the right hand side are first.

Solutions for Problem 2 are found in Section 4.2 of our textbook. (Solutions for Problem 1 will be uploaded.)