MAT 248B: PROBLEM SET 2

DUE TO MONDAY MAY 8TH

ABSTRACT. This is the second problem set of the course MAT-248B Spring 2026. It is due Friday May 8 at 3:00pm submitted via Gradescope.

Task: Solve one of the problems below and submit it through Gradescope by Friday May 8th at 3pm. Be rigorous and precise in writing your solutions.

Problem 1. Consider the affine subvariety $Y = \{x^5 = y^3\} \subseteq \mathbb{C}^2$ and its coordinate ring

$$\mathcal{O}_Y(Y) \cong \mathbb{C}[x, y]/(x^5 - y^3).$$

(a) Describe the cotangent sheaf Ω_Y as a sheaf $\mathcal{O}_Y$-module.

(b) At each point of Y, compute the dimension of the stalk $(\Omega_Y)_p$ and find a basis of $(\Omega_Y)_p$ as an $\mathcal{O}_{Y,p}$-module.

(c) Show that the conormal sheaf of Y in $\mathbb{C}^2$ is locally free.

Problem 2. Consider the projective curve $\{x^3 - yz^2 = 0\} \subseteq \mathbb{P}^2_k$. Find the cohomologies of its tangent sheaf and its normal sheaf.

Problem 3. Give an instance of an affine subvariety $Y \subseteq \mathbb{C}^n$ such that the conormal sheaf is *not* locally free.

Problem 4. Show that the sheaf $\mathcal{O}_{\mathbb{P}^1}(n)$ on $\mathbb{P}^1$ is coherent for any $n \in \mathbb{Z}$.

Problem 5. Let C be a projective curve and $f : C \longrightarrow \mathbb{P}^1$ a given finite morphism of degree d (e.g. a degree d simple branched cover). Compute the cohomology of $\mathcal{O}_C$ using f.

Problem 6. Consider the projective surface $X = \mathbb{P}^1_k \times \mathbb{P}^1_k$, compute $H^*(X, \mathcal{O}_X)$ using Cech cohomology.

Problem 7. Compute the cohomology of the structure sheaf $\mathcal{O}_X$ of the algebraic variety $X = \mathbb{A}^n_k \setminus \{(0, \dots, 0)\}$, i.e. X is affine n-space with the origin removed, $n \geq 2$. Conclude that X is not affine.

Problem 8. Exercise 4.9 from Hartshorne's Chapter III.

Problem 9. Give an instance of a (necessarily singular) projective variety that does *not* admit a dualizing sheaf.