University of California Davis **Name (Print):** ______________________
Algebraic Topology II MAT 215B **Student ID (Print):** ______________________

Midterm Examination **February 7 2020**
Time Limit: 50 Minutes

This examination document contains 6 pages, including this cover page, and 4 problems. You must verify whether there any pages missing, in which case you should let the instructor know. **Fill in** all the requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

(A) **If you use a lemma, proposition or theorem which we have seen in the class or in the book, you must indicate this** and explain why the theorem may be applied.

(B) **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive little credit.

(C) **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive little credit; an incorrect answer supported by substantially correct calculations and explanations will receive partial credit.

(D) If you need more space, use the back of the pages; clearly indicate when you have done this.

Problem	Points	Score
1	25	
2	25	
3	25	
4	25	
Total:	100	

Do not write in the table to the right.

1. (25 points) Let X be a connected CW-complex and $S^1 = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}$ the unit circle. Endow the Cartesian product $X \times S^1$ with its product topology.
   (a) (15 points) Show that $H_k(X \times S^1) = H_{k-1}(X) \oplus H_k(X)$ for all $k \geq 0$.

   (b) (10 points) Compute the homology $H_*(T^4)$ of the 4-torus $T^4 = S^1 \times S^1 \times S^1 \times S^1$.

2. (25 points) Let Z be a path-connected CW-complex, and consider

$$Y = \{(z_1, z_2) \in \mathbb{C}^2 : z_1^3 + z_2^5 = 1\} \subseteq \mathbb{C}^2,$$

$$X_t = \{(z_1, z_2, z_3) \in \mathbb{C}^3 : z_1^3 + z_2^5 + z_3^2 = t\} \subseteq \mathbb{C}^3, t \in [0, 1].$$

(a) (5 points) Compute the homology groups $H_*(X_0)$.

(b) (5 points) Show that $Y \simeq \bigvee_{i=1}^{8} S^1$ and compute $H_*(Y)$.

(c) (5 points) Prove that $\widetilde{H}_{*+1}(\Sigma Z) \cong \widetilde{H}_*(Z)$, $* \geq -1$, where $\Sigma Z = (Z \times [0,1])/\sim$ is the *unreduced* suspension of Z.

(d) (10 points) Compute the homology groups $H_*(X_t)$, for all $t \in (0,1]$.

3. (25 points) Consider the following commutative diagram of Abelian groups, where the two rows are exact:

$$\begin{array}{ccccccccc}
A & \xrightarrow{f} & B & \xrightarrow{g} & C & \xrightarrow{h} & D & \xrightarrow{j} & E \\
\downarrow{\scriptstyle l} & & \downarrow{\scriptstyle m} & & \downarrow{\scriptstyle n} & & \downarrow{\scriptstyle p} & & \downarrow{\scriptstyle q} \\
A & \xrightarrow{r} & B & \xrightarrow{s} & C & \xrightarrow{t} & D & \xrightarrow{u} & E
\end{array}$$

Show that n is injective if m, p are injective and l is surjective.

4. (25 points) The topological spaces X, Y, A, B are path-connected CW-complexes. For each of the sentences below, circle **true** or **false**. You do *not* need to justify your answer.

(a) (2 points) An injective continuous map $f : X \longrightarrow Y$ between topological spaces induces an injective map $H_*(f) : H_*(X) \longrightarrow H_*(Y)$.

(1) True. (2) False.

(b) (2 points) If X admits a cell decomposition with a unique top-dimensional cell $e_n : \Delta^n \longrightarrow X$, then $H_n(\Sigma) \cong \mathbb{Z}$.

(1) True. (2) False.

(c) (2 points) If $H_*(X) \cong H_*(Y)$ are isomorphic as Abelian groups then it must be that X is homotopy equivalent to Y.

(1) True. (2) False.

(d) (2 points) Suppose that $f, g : (A_*, \partial^A) \longrightarrow (B_*, \partial^B)$ are chain maps such that $H_*(f) = H_*(g)$. Then $f \simeq g$ are chain homotopic.

(1) True. (2) False.

(e) (2 points) Let $A, B \subseteq X$ be topological subspaces. Then $H_*(X, A) \cong H_*(B, A \cap B)$.

(1) True. (2) False.

(f) (2 points) A surjective continuous map $f : X \longrightarrow Y$ between topological spaces induces a surjective map $H_*(f) : H_*(X) \longrightarrow H_*(Y)$.

(1) True. (2) False.

(g) (2 points) If X admits a cell decomposition with a unique zero-dimensional cell $e_0 : \Delta^0 \longrightarrow X$, then $H_0(X) \cong \mathbb{Z}$.

(1) True. (2) False.

(h) (2 points) Suppose that $\pi_1(X) \cong \pi_1(Y)$. Then $H_1(X) \cong H_1(Y)$.

(1) True. (2) False.

(i) (2 points) Let $X \subseteq \mathbb{R}^n$ and $Y \subseteq \mathbb{R}^m$ be two open sets. If X and Y are homotopy equivalent, then $n = m$.

(1) True. (2) False.

(j) (2 points) Let $K \subseteq S^3$ be a knot, then $H_*(S^3 \setminus K)$ is independent of K.

(1) True. (2) False.