Computational Fluid Dynamics of Crossflow Filtration in Suspension-Feeding Fishes

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Abstract
Suspension-feeding fishes such as herring and anchovies engulf particle-concentrated water through their mouths and release the water through the posterior oral cavity. Food particles are separated from the water at the gill rakers that act like modern crossflow filters. This paper uses Computational Fluid Dynamic (CFD) techniques to study the feeding mechanism of these suspension feeders. By understanding how food is separated from the water, we can elucidate why fish gill rakers do not get clogged with particles in the same manner that industrial crossflow filters eventually become fouled.\(^*\)

Introduction
Suspension-feeding fishes filter small particles from enormous volumes of water that enter their mouths. These fishes, belonging to 21 families in 12 orders, are critical components of many ecological communities [1] and comprise approximately 25% of the annual world fish catch [2]. As the details of oral cavity structure are very complex, we focus on the morphological features that are generally considered to serve the most critical functions during suspension feeding [3]. During suspension feeding, water that enters the mouth travels posteriorly to exit the oral cavity via branchial slits between the gill arches on both sides of the head. The gill rakers are finger-like projections of the gill arch on the opposite side from the gill filaments. Figure 1 shows the arrangement of the gill rakers and gill arch of a bony fish [4].

In this paper, CFD techniques are used to study the feeding mechanism of suspension-feeding fishes [5-6]. Gill rakers have been postulated to function as a `dead-end' filter, collecting small food items by sieving [7] or by hydrosol filtration, in which particles smaller than the size of the filter pores stick to the filter's elements [8]. A potential problem with dead-end or hydrosol filtration, however, is that the filter gets clogged as tiny food particles become trapped on the gill rakers. Moreover, Sanderson, et al. [9] observed that more than 95% of food particles do not come into contact with the gill rakers but instead remain suspended in fluid flows parallel to the filter surface as water leaves between the gill rakers. Particles become more concentrated as they travel towards the esophagus, and the fish swallows concentrated slurry of food particles. Thus, the gill rakers act like a modern crossflow filtration [10] mechanism similar to those in our water treatment plants. However, unlike most industrial crossflow filtration, particles do not accumulate on the gill rakers. Sanderson et al.[9] estimated the value for one theoretical transport mechanism--- radial inertial migration --- and found that it is inadequate, by an order of magnitude, to explain

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particle transport in fishes that feed by crossflow filtration. The underlying physical mechanisms are not yet fully understood. Our goal is to investigate the underlying mechanism of crossflow filtration in suspension feeders.

In this study, a 3-D Navier-Stokes flow solver is used to calculate the flow field around the fish rakers and small spherical food particles are released into the flow field so that we can track the particle trajectories. This enables us to compare the effects of different flow conditions (speed and incident angles) on particles of different size and mass inside the oral cavity of the fish. For these calculations, we assume Newtonian flow since the particles we are simulating are small enough that they do not influence the flow field.

**Overflow**

In this study, the OVERFLOW code developed at NASA is used to solve the Navier-Stokes (N-S) equations for structured Chimera overset grids (see Figure 2). This code is an outgrowth of the previous computer codes named ARC3D [11] and F3D [12] which have various options of numerical algorithms. Local time step scaling, grid sequencing and multi-grid are also implemented for convergence acceleration. Our calculations require the Low-Mach number preconditioning technique where the resulting matrix equation is solved using the Pulliam-Chaussee diagonalized (scalar pentadiagonal) scheme [13] and the Roe upwind scheme [14].

**Particle Traces Algorithm**

The following algorithm is used to trace the trajectory of food particles that enter the oral cavity of the fish. The velocity field $V(x)$ is obtained from solving the N-S equations and the initial position of a particle $x(0)$ is prescribed. The particle path $x(t)$ is then obtained by first solving the following equation $\frac{dx}{dt} = v(x)$.

Discretizing this equation using the second-order Runge-Kutta integration scheme with adaptive step sizing [15], we get

$$x^* = x^k + dtv(x^k)$$
$$x^{k+1} = x^k + dt\frac{1}{2}(v(x^k) + v(x^*))$$

(1)

The value of $dt$ is given by $dt = c / \max(V_x, V_y, V_z)$; $V_x$, $V_y$, and $V_z$ are the three components of the velocity $v$; and $0 < c < 1$. The constant $c$ is used for adaptive step sizing and controls the number of steps that $x$ will advance in each grid cell. Adaptive step sizing is required when the velocity varies rapidly in some regions of the grid. The algorithm described above assumes that the flow is steady (time-independent).

When the particle has finite mass, additional formulas are needed due to Newton's Second Law. Let $m$ be the mass of the particle and let $F(v)$ be the force applied to the particle.

Newton's Second Law states that $\frac{dv}{dt} = \frac{1}{m}F(v)$.

Again using the second-order Runge-Kutta integration scheme, we have

$$v^* = v^{k-1} + dtF(v^{k-1})/m$$
$$v^k = v^{k-1} + dt\frac{1}{2}(F(v^{k-1}) + F(v^*))/m$$

(2)
Thus for particle with finite mass, the trajectory (path) is obtained by combining (1) and (2). The values of \( v(x^k) \) and \( v(x^*) \) in (1) are replaced by \( v^k \) and \( v^* \) in (2) respectively.

The forces applied to a moving particle of velocity \( v \) at a given instant are the sum of the buoyant force, gravitational force, and drag. Thus, the force vector \( F \) is expressed by

\[
F(v) = -V\rho_w g z + V\rho_p g z + C_D \frac{1}{2} \rho_w S |w|^2 \hat{w}
\]  

(3)

where \( V \) is the volume of the particle, \( g \hat{z} \) is the gravitation acting on in the \( \hat{z} \) direction, \( \rho_w \) is the water density, and \( \rho_p \) is the particle density. The drag force is a function of water density \( (\rho_w) \), particle wetted area \( (S) \), drag coefficient \( (C_D) \) of the particle in water, and the traveling velocity \( (w, \text{ is the fluid velocity } u \text{ minus particle velocity } v; \hat{w} \text{ is the unit vector}) \. The mass \( (m) \) of the particle is \( V\rho_p \), thus

\[
F(v) = (1 - \frac{\rho_w}{\rho_p})g \hat{z} + \frac{C_D}{2} \frac{\rho_w S |w|^2}{m} \hat{w}
\]

(4)

Note that if the particle density is the same as the water density, the buoyancy and the gravitation terms cancel and the first term in (4) is zero.

In order to calculate the particle trace under the influence of the flow pattern \( v(t) \), we need to calculate the drag coefficient \( (C_D) \). To do this, consider a spherical particle dropped into a tank of water. It will reach a constant velocity, called terminal velocity \( (v_t) \). At terminal velocity, the submerged weight of the particle is the same as the drag \( (D) \) on the particle. The submerged weight is no more than the difference of buoyancy and gravity; that is the first two terms in (3). Therefore, from (3), \( V\rho_p g - V\rho_w g = D \).

Since the food particles are very small (~100 microns) and the Reynolds number is very low (<0.2), we can use the well-known Stokes relation for drag [16], \( D = 6\pi \mu r \). Here \( r \) is the radius of the sphere and \( m \) is the absolute viscosity of the fluid. Therefore,

\[
C_D = \frac{1}{\frac{1}{2} \rho_w S V^2_t} = \frac{V\rho_p g - V\rho_w g}{\frac{1}{2} \rho_w S V^2_t} = \frac{6\pi \mu r}{\frac{1}{2} \rho_w S V g (\rho_p - \rho_w)}
\]

(5)

The Simulation
To simulate flow through the rakers of a generalized suspension-feeding fish, the velocity field is calculated through an infinite array using periodic boundary conditions on a finite array of elliptical cylinders (Fig. 2). Each elliptical cylinder has dimensions 375 microns in major axis diameter, 250 microns in minor axis diameter. The gap between two cylinders is 250 microns. The suspended food particles are represented in our calculations by spheres with radius 82.5 microns \( (8.25 \times 10^{-5} \text{ m}) \) and density \( (\rho_p) \) is 1.05gm/cc \( (1050 \text{kg/m}^3) \). The water density \( (\rho_w) \) and viscosity \( (m) \) are 1000kg/m\(^3\) and 0.001kg/ms respectively. The gravitation constant \( (g) \) is 9.81, the volume \( (4/3)\pi r^3 \) and the wetted area of the \( \pi r^2 \). With this information, the drag coefficient of the particles can be obtained from Eqn. 5.
Computation Results
Figures 3a and b show a 2-D view of a 3-D calculation of flow at 0.6m/s with incident angle 75 and 85 degrees, respectively for an infinite array. The upper figures show the velocity contours and the lower figures show the streamlines. The vortex between the rakers in Fig. 3b suggests that it may be an impediment to particle movement between the rakers in contrast to Fig. 3a where the center of the large vortex is not in the gap.

The differences in vortex structure at the two different incident angles are consistent because complex vortex structures are always created in the aft area of an obstacle and the circled areas in Fig. 3 in front of the leading edge of the ellipse indicate the highest velocity region.

When a particle with density $\rho_p = 1.05\text{gm/cc}$ is introduced into the flow, its trajectory deviates from the streamlines as shown in Fig. 4 and it tends to keep a larger distance from the leading edge of the raker. This means that the particle tends to travel with the high velocity flow rather than escaping through the gap between rakers. Examination of the velocity vector shows a very slow recirculating flow inside the gap between the rakers. In contrast, the flow in front of the rakers accelerates to 0.682 m/s which is higher than the free-stream velocity of 0.6 m/s at the leading edge of the rakers. Since the force applied to a particle is tangential to the velocity vector and since there is a low circular velocity field in the gap parallel to the high velocity flow outside, food particles will not cross high flow regions to escape between the gaps. Thus, although the gap is large (250 $\mu$m) compared to the particle size, the effective gap is small, only about 50$\mu$m, indicating that particles larger than $1/5$ of the gap will not tend to escape.

Following the trajectory of two particles starting at the same position, one massless particle that follows the streamlines and one with density $\rho_p = 1.05\text{gm/cc}$ that deviates from the streamlines, we note that food particles with drag will travel further downstream (Fig. 5). This is consistent with the hypothesis that the gill rakers act like a crossflow filter rather than a dead-end filter and with the observations of Sanderson et al. [9].

Reference:

Fig. 1 Structure of gill rakers and gill arches: (a) the position of gill arches inside the gill cavity; (b) a side view of the first gill arch. (Miler and Lea 1972)

Fig. 2 The Computational Grids.

Fig. 3a, b Steady flow solutions with incident angle of 75 and 85 degrees

Fig. 4 A side-by-side comparison of particle traces of massless particles (a) and finite-weight particles (b). The incident angle is 85 degrees

Fig. 5. Flow with speed 60cm/s. (a) A massless particle is released at a point upstream; it passes between 2nd and 3rd rakers. (b) A particle with drag is released at the same physical point; it travels further downstream before exiting between 3rd and 4th rakers