1. Peer-reviewed publications, annotated

(1) *Realizable sets of catenary degrees of numerical monoids*,
with Roberto Pelayo.

The catenary degree is an invariant that measures the distance between factorizations of elements within an atomic monoid. We classify which finite subsets of $\mathbb{Z}_{\geq 0}$ occur as the set of catenary degrees of a numerical monoid. (arXiv:1705.04276)

(2) *Unimodular hierarchical models and their Graver bases*,
with Daniel Bernstein.

Given a simplicial complex whose vertices are labeled with positive integers, one can associate a vector configuration whose corresponding toric variety is the Zariski closure of a hierarchical model. We classify all vertex-weighted simplicial complexes that give rise to unimodular vector configurations, and provide a combinatorial characterization of their Graver bases. (arXiv:1704.09018)

(3) *On the periodicity of irreducible elements in arithmetical congruence monoids*,
with Jacob Hartzer.
To appear, Integers.

In this paper, we examine the asymptotic behavior of the set of irreducible elements of arithmetical congruence monoids, and characterize when this set forms an eventually periodic sequence. (arXiv:1606.00376)

This is an undergraduate research project from Texas A&M University.

(4) *On factorization invariants and Hilbert functions*.

In the setting of finitely generated semigroups, several factorization invariants, including the delta set, $\omega$-primality, and catenary degree, are expressed in terms of Hilbert functions of multigraded modules. Consequently, several recent results for numerical semigroups are recovered, and each is extended to finitely generated semigroups. (arXiv:1503.08351)

(5) *Mesoprimary decomposition of binomial submodules*.

Mesoprimary decomposition, a combinatorial method for obtaining primary decompositions of binomial ideals, is generalized to “binomial submodules” of certain graded
modules over a monoid algebra, analogous to the way primary decomposition of ideals over a Noetherian ring $R$ generalizes to $R$-modules. (arXiv:1511.00161)


We present structural results on solutions to the Diophantine system $Ay = b, y \in \mathbb{Z}_{\geq 0}$ with the smallest number of non-zero entries, and discuss some interesting consequences in discrete optimization. Our tools are algebraic and number theoretic in nature and include Siegel’s Lemma, generating functions, and commutative algebra. (arXiv:1602.00344)


This survey article gives an overview of the length set, elasticity, delta set, $\omega$-primality, and catenary degree invariants in the setting of numerical monoids. For each invariant, we present current major results in the literature and identify the primary open questions that remain. (arXiv:1508.00128)


We investigate the benefits of dynamic programming when computing factorization invariants in numerical monoids. We give dynamic programming algorithms to compute the delta set and $\omega$-primality of any numerical monoid element. (arXiv:1507.07435)

This is an undergraduate research project coadvised with Roberto Pelayo.


We show that the set of length sets for any arithmetical numerical monoid can be completely recovered from its set of elasticities. Additionally, we prove a structure theorem for the set of elasticities of any numerical monoid. (arXiv:1409.3425)

This is an undergraduate research project coadvised with Roberto Pelayo.


We construct irreducible decompositions of binomial ideals by introducing the notion of a socular monoid congruence; we demonstrate that some binomial ideals do not admit binomial irreducible decompositions. (arXiv:1503.02607)

We provide structural results on the set of catenary degrees achieved in finitely generated cancellative commutative monoids, including a computable lower bound in terms of the monoid’s Betti elements. (arXiv:1506.07587)

This is an undergraduate research project from the SDSU Mathematics REU in 2014, coadvised with Vadim Ponomarenko.


We provide an explicit formula for several factorization invariants of numerical monoids generated by geometric sequences. (arXiv:1503.05993)

This is an undergraduate research project from the SDSU Mathematics REU in 2014, coadvised with Vadim Ponomarenko.


We study the factorization theory of Leamer monoids, and compute several factorization invariants. (arXiv:1309.7477)

This is an undergraduate research project from the PURE Mathematics REU in 2013, coadvised with Roberto Pelayo.


The $\omega$-function measures how far a semigroup element is from being prime. We prove that the $\omega$-function for any numerical semigroup is eventually quasilinear. (arXiv:1309.7476)


Extensions of the mesoprimary framework set forth by Kahle and Miller are presented.
2. Submitted preprints, annotated

(1) *Random numerical semigroups and a simplicial complex of irreducible semigroups*,
with Jesus De Loera and Dane Wilbourne.
Submitted.

We examine properties of random numerical semigroups under a probabilistic model inspired by the Erdős-Rényi model for random graphs. We provide a threshold function for cofiniteness, and bound the expected embedding dimension, genus, and Frobenius number of semigroups generated with this model. ([arXiv:1710.00979](https://arxiv.org/abs/1710.00979))

(2) *Factoring in the Chicken McNugget monoid*,
with Scott Chapman.
Submitted to Mathematics Magazine.

We present an accessible introduction to the Chicken McNugget problem: “what numbers of Chicken McNuggets can be ordered using only packs with 6, 9, or 20 pieces?” We also discuss several related questions whose motivation comes from the theory of non-unique factorization. ([arXiv:1709.01606](https://arxiv.org/abs/1709.01606))

(3) *Apéry sets of shifted numerical monoids*,
with Roberto Pelayo.
Submitted.

We give a highly efficient algorithm for computing Apéry sets of numerical semigroups obtained by “shifting” the minimal generators of a given numerical semigroup, and prove that several numerical monoid invariants, such as the genus and Frobenius number, are eventually quasipolynomial as a function of the shift parameter. ([arXiv:1708.09527](https://arxiv.org/abs/1708.09527))

(4) *On mesoprimary decomposition of monoid congruences*.
Submitted.

We prove two main results concerning mesoprimary decomposition of monoid congruences, which is the first step in a combinatorial method for obtaining primary decompositions of binomial ideals. First, we identify which associated prime congruences appear in every mesoprimary decomposition, thereby completing the theory of mesoprimary decomposition of monoid congruences as a more faithful analog of primary decomposition. Second, we characterize which finite posets can arise as the set of associated prime congruences of a monoid congruence. ([arXiv:1708.03441](https://arxiv.org/abs/1708.03441))

(5) *A computer algebra system for R: Macaulay2 and the m2r package*,
with David Kahle and Jeff Sommars.
Submitted.

We debut m2r, an R package that connects R to Macaulay2 through a persistent back-end socket connection running locally or on a cloud server. Topics range from basic use of m2r to applications and design philosophy. ([arXiv:1706.07797](https://arxiv.org/abs/1706.07797))
(6) *Some algebraic aspects of mesoprimary decomposition*,
with Laura Matusevich.
Submitted.

We examine mesoprimary decomposition, a combinatorial method for obtaining primary decompositions of binomial ideals, in the presence of a positive $A$-grading, where certain pathologies are avoided and the theory becomes more accessible. (arXiv:1706.07496)

(7) *The elasticity of Puiseux monoids*,
with Felix Gotti.
Submitted.

We examine the elasticity invariant for Puiseux monoids (additive submonoids of $\mathbb{Q}_{\geq 0}$). We give a formula, in terms of the atoms, for the elasticity of any Puiseux monoid, and classify when the elasticity is accepted. We also obtain the set of elasticities for a special class of Puiseux monoids, and provide an example of a bifurcus Puiseux monoid (that is, every reducible element has a length 2 factorization). (arXiv:1703.04207)

(8) *Minimal presentations of shifted numerical monoids*,
with Rebecca Conaway, Felix Gotti, Jesse Horton, Roberto Pelayo, Mesa Williams, and Brian Wissman.
Submitted.

We investigate numerical semigroups obtained by “shifting” the minimal generators of a given numerical semigroup. More specifically, we examine minimal relations among the generators when the shift parameter is sufficiently large, culminating in a description that is periodic in the shift parameter. We also explore several applications to computation, combinatorial commutative algebra, and factorization theory. (arXiv:1701.08555)

This is an undergraduate research project from the PURE Mathematics REU in 2015, coadvised with Roberto Pelayo and Brian Wissman.

(9) *On the computation of factorization invariants for affine semigroups*,
with Pedro García-Sánchez and Gautam Webb.
Submitted.

In the setting of affine semigroups, we present the first known algorithm for computing the delta set, an improved algorithm for computing the tame degree, and a dynamic algorithm for computing the catenary degree of semigroup elements. (arXiv:1504.02998)