Given below are the required problems for this assignment. Please submit your answers on a printed copy of this sheet. For this assignment, you may omit either Exercise 3 or Exercise 4.

(1) Solve the recurrence relation $a_n = 7a_{n-1} - 12a_{n-2} + 5^n$, $a_0 = 1$, and $a_1 = 1$. 
(2) Let $a_n = \sum_{i=0}^{n} i$. Notice that we can (equivalently) define $a_n$ as a recurrence relation $a_n = a_{n-1} + n$ for $n \geq 1$ and $a_0 = 0$. Solve this recurrence relation, and compare the resulting formula to our usual closed form $n(n+1)/2$ for $\sum_{i=0}^{n} i$. 
(3) Use generating functions to find the general solution to the recurrence relation

\[ a_n = 12a_{n-1} - 35a_{n-2}, \quad a_0 = 0, a_1 = 2. \]
(4) Prove that the set \( \mathbb{Z}^2 \) is countable (Hint: use a picture or diagram to illustrate how one might enumerate the elements of this set).