(Q1) Fix a finite dimensional $k$-vector space $V$ and a finite set of vectors $E \subset V$, and let

$$\mathcal{I}(E) = \{ I \subset E : I \text{ contains no linear dependencies}\}.$$ 

Prove that $(E, \mathcal{I}(E))$ forms a matroid. Such matroids are called linear.

(Q2) Fix a finite graph $G$, and let $E = E(G)$ denote the set of edges, and let

$$\mathcal{I}(G) = \{ I \subset E : I \text{ contains no cycles}\}.$$ 

Prove that $(E, \mathcal{I}(E))$ forms a matroid. Such matroids are called graphical.

(Q3) Fix a bipartite graph $G = (A, E)$, and let

$$\mathcal{I}(E) = \{ I \subset E : |N(I)| = |I|\},$$

where $N(I) \subset A$ denote the set of neighbors of vertices in $I$. Prove that $(E, \mathcal{I}(E))$ forms a matroid. Such matroids are called transversal.

(Q4) Prove that every graphical matroid is linear.