Math 16B: Short Calculus II  
Spring 2017, Section 1  
Homework Sheet 4  
Due: Monday, May 1, 2017

Submit your solutions to the following problems in lecture on the due date above. Present your work in a clean and organized fashion, either on a printed copy of this document (preferred) or a separate sheet of paper. As stated in the syllabus, late submissions will not be accepted.

1. Evaluate the following integrals.

   (a) \[ \int_0^\pi 21 \sin(7x) \, dx = \left[ -3 \cos(7x) \right]_0^\pi = \left( -3 \cos(7\pi) \right) - \left( -3 \cos(0) \right) = 3 - (-3) = 6 \]

   (b) \[ \int_{\ln(\pi/2)}^{\ln(\pi)} e^x \cos(e^x) \, dx = \sinh(e^x) \bigg|_{\ln(\pi/2)}^{\ln(\pi)} = \sinh(e^{\ln(\pi)}) - \sinh(e^{\ln(\pi/2)}) = \sinh(\pi) - \sinh(\pi/2) = 0 - 1 = -1 \]

2. Consider the following integral.

   \[ \int_0^\pi \sin(x) \, dx \]

   (a) Approximate the above integral using a midpoint sum with \( n = 3 \) subdivisions.

   \[ \text{Estimate} = \sinh\left(\frac{\pi}{6}\right) \cdot \left( \frac{\pi}{3} \right) + \sinh\left(\frac{\pi}{2}\right) \cdot \left( \frac{\pi}{3} \right) + \sinh\left(\frac{\pi}{6}\right) \cdot \left( \frac{\pi}{3} \right) \]

   \[ = \left( \frac{1}{2} \right) \left( \frac{\pi}{3} \right) + \left( 1 \right) \left( \frac{\pi}{3} \right) + \left( \frac{1}{2} \right) \left( \frac{\pi}{3} \right) = \frac{2 \pi}{3} \]

   (b) Compare each of your estimated to the exact area under the curve.

   \[ \int_0^\pi \sin(x) \, dx = -\cos(x) \bigg|_0^\pi = (-\cos(\pi)) - (-\cos(0)) = 1 - (-1) = 2 \]

   \[ \text{Actual} = 2 \]

   \[ \text{Estimate} = \frac{2 \pi}{3} \approx 2.094 \]

\[ \frac{\pi}{6} \]

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