The problems below are intended to help you review for the final exam, and may not be turned in for credit.

(ER1) Determine whether or not each of the following block designs exists. For those that do, specify a method to construct one and find the number of blocks it would have. If the number of blocks is at most 10, find the design explicitly.

(a) A 2-design (13, 4, 1).
(b) A 2-design (49, 17, 1).
(c) A 2-design (10201, 101, 1).
(d) A 1-design (6, 4, 5).
(e) A 1-design (6, 4, 6).
(f) A 7-design (17, 17, 1).

(ER2) Consider the set \( A = \{0, 1, 2, 4\} \subset \mathbb{Z}_7 \).

(a) Verify that \( A \) is a difference set.
(b) Construct a 2-design using \( A \), and specify its parameters.
(c) Find the complementary 2-design to the design in part (b). Specify its parameters.
(d) View your designs in parts (b) and (c) as 1-designs and find their parameters.

(ER3) Determine which of the following codes are linear, and find the value of \( \delta \) for each.

(a) \( \{000000, 011010, 101101, 110011\} \subset V_8^6 \).
(b) \( \{000000, 011010, 100101, 111111\} \subset V_8^6 \).
(c) \( \{000000, 100000, 010000, 001000, 000100, 000011\} \subset V_5^5 \).

(ER4) What is the maximum dimension of a linear code \( C \subset V_8^8 \) that can correct 2 errors? Find a code with this property (you may give a basis, specify a check matrix, or simply list all of the codewords).

(ER5) Consider the code \( C \) defined by the following check matrix.

\[
\begin{bmatrix}
1 & 0 & 1 & 0 & 1 & 1 \\
0 & 1 & 0 & 0 & 1 & 0 \\
1 & 1 & 0 & 0 & 0 & 1 \\
1 & 0 & 1 & 1 & 0 & 1
\end{bmatrix}
\]

(a) Find a basis for \( C \).
(b) Determine whether \( C \) is guaranteed to correct at least one error.
(c) Suppose you receive a code 11111111. Is this a valid codeword in \( C \)? If not, which codeword(s) can it correct to?

(ER6) Is the set \( R = \{x^n + a_{n-1}x^{n-1} \cdots + a_1x + a_0 : a_0, \ldots, a_{n-1} \in \mathbb{Q}\} \subset \mathbb{Q}[x] \) a ring? If so, is it a field?

(ER7) Determine whether the polynomial \( x^3 + 5x^2 + 3x + 4 \in \mathbb{Q}[x] \) is irreducible.

(ER8) True or false: there is more than one field with infinitely many elements.

(Bonus) Find all codewords for the cyclic code \( C \subset V_4^4 \) corresponding to the ideal \( \langle x + 1 \rangle \).