Math 16B: Short Calculus II  
Winter 2018, Section 3  
Homework Sheet 4  
Due: Monday, February 12, 2018

Submit your solutions to the following problems in lecture on the due date above. Present your work in a clean and organized fashion, either on a printed copy of this document (preferred) or a separate sheet of paper. As stated in the syllabus, late submissions will not be accepted.

1. Evaluate the following integrals.

(a) \[ \int_3^6 (x^2 + 2x + 1) \, dx = \left. \frac{1}{3} x^3 + X^2 + x \right|_{x=3}^{x=6} = \left( \frac{1}{3} (6)^3 + (6)^2 + 6 \right) - \left( \frac{1}{3} (3)^3 + (3)^2 + 3 \right) = 114 - 21 = 93 \]

(b) \[ \int_0^\pi 21 \cos(7x) \, dx = 21 \sin(7x) \bigg|_{x=0}^{x=\pi} = 21 \sin(7\pi) - 21 \sin(0) = 0 \]

\[ \text{Hence, } \frac{1}{3} \left( \int_0^\pi \sin(7x) \, dx \right) = 7 \cos(7x) \bigg|_{x=0}^{x=\pi} = 21 \cos(7\pi) \]

2. Consider the following integral.

\[ \int_0^\pi \sin(x) \, dx \]

(a) Approximate the above integral using a midpoint sum with \( n = 3 \) subdivisions.

\[ \left[ \sin \left( \frac{\pi}{6} \right) \right] \cdot \frac{\pi}{3} + \left[ \sin \left( \frac{\pi}{2} \right) \right] \cdot \frac{\pi}{3} + \left[ \sin \left( \frac{5\pi}{6} \right) \right] \cdot \frac{\pi}{3} = \left( \frac{1}{2} \right) \left( \frac{\pi}{3} \right) + \left( 1 \right) \left( \frac{\pi}{3} \right) + \left( \frac{1}{2} \right) \left( \frac{\pi}{3} \right) = \frac{2\pi}{3} \]

(b) Compare your estimate to the exact area under the curve.

\[ \int_0^{\pi} \sin(x) \, dx = -\cos(x) \bigg|_{x=0}^{x=\pi} = \left( -\cos(\pi) \right) - \left( -\cos(0) \right) = 1 + 1 = 2 \]

\[ \text{Error} = \left| 2 - \frac{2\pi}{3} \right| \]