MATH 22AL
Lab # 2

1 Objectives

In this LAB you will explore the following topics using MATLAB.

- Matrix Operations
- Symmetric and Skew-Symmetric Matrices
- Using MATLAB to solve linear systems

The first two pages of this lab, is a summery of the general instructions in doing the LABS.

2 Recording and submitting your work

The following steps will help you to record your work and save and submit it sucsessfuly.

- Open a terminal window.
  - In 2118 MSB click on terminal Icon at the bottom of the screan
  - Windows OS Use Putty
  - MAC OS Use terminal window of MAC.
- Start a MATLAB Session that is :
  - Type "textmatlab" Press Enter
  - Type "diary LAB2.text" Press Enter
- Enter your information that is :
  - Type "% Last Name:" then enter your Last name
  - Type "% First Name:" then enter your first name
  - Type "% Date:" then enter the date
  - Type "% Username:" then enter your Username for 22AL account
- Do the LAB that is :
  - Follow the instruction of the LAB.
  - Type needed command in MATLAB.
  - All commands must be typed in front of MATLAB Command " >> " .
- Close MATLAB session Properly that is :
  - When you are done or if you want to stop and continue later do the following:
  - Type "save" Press Enter
  - Type "diary off" Press Enter
  - Type "exit" Press Enter
- Edit Your Work before submitting it that is :
  - Use pico or editor of your choice to clean up the file you want to submit:
  - in command line of point (or any of the math department computers) type "pico LAB2.text"
  - Delete the errors or insert missed items.
  - Save using "^ o= control key then o"
  - Exit using "^ x= control key then x"
- Send your LAB that is :
  - Type "ssh point" : Press enter
  - Type submitm22al LAB2.text

LAB 2 Starts Please make sure you have started MATLAB and has typed diary LAB2.text

- Type "% Last Name:" then enter your Last name
- Type "% First Name:" then enter your first name
- Type "% Date:" then enter the date
- Type "% Username:" then enter your Username for 22AL account

3 Format

Default Display
By default, MATLABÅ¡ displays numeric output as 5-digit scaled, fixed-point values. You can change the way numeric values are displayed to any of the following:

- 5-digit scaled fixed point, floating point, or the best of the two
- 15-digit scaled fixed point, floating point, or the best of the two
- A ratio of small integers
- Hexadecimal (base 16)
- Bank notation

Please note: :

- The format function changes the display of numeric values for the duration of a single MATLAB session, while your Preferences settings remain active from one session to the next.
- These settings affect only how numbers are displayed, not how MATLAB computes or saves them.

Type $A1 = [2/3\ 4/3\ 5/2]$
What you see is the default format of MATLAB, how it displays the numerical values, you can change this using the format command (function) as in the following examples:
Type format rat
Type $A1 = [2/3\ 4/3\ 5/2]$
Type format short
Type $A1 = [2/3\ 4/3\ 5/2]$
Type format long
Type $A1 = [2/3\ 4/3\ 5/2]$
Type fix(A1)
This will round the entries of A to nearest zero.
Type help format

4 Extracting Triangular Matrices

Create a lower triangular matrix by typing :
$A = [1\ 2\ 3\ 4;\ 4\ 5\ 6\ 5;\ 6\ 6\ 6\ 5;\ 2\ 1\ 4\ 7]$
$B1 = tril(A)$
$B2 = tril(A, 1)$
$B3 = tril(A, -1)$
$B4 = tril(A, -2)$
$B5 = tril(A, 0)$
$B6 = tril(A, 2)$

Try to guess the role of the integer number in the second component. To learn the details,you may type
Now type the following: $C1 = triu(A)$
$C2 = triu(A, 1)$
$C3 = triu(A, -1)$
$C4 = triu(A, -2)$
$C5 = triu(A, 0)$
$C6 = triu(A, 1)$
$C7 = triu(A, 2)$

To learn more about these two functions which Extract lower or upper triangle from input matrices you may visit the following :

http://www.mathworks.com/help/matlab/ref/tril.html?searchHighlight=tril
http://www.mathworks.com/help/matlab/ref/triu.html?searchHighlight=triu

5 Matrix operations

Enter B=[2 2 2 2 ; 3 3 4 3; 5 5 1 1 ; 2 -1 2 0]
and
C=ones(4),

recall that $A = [1\ 2\ 3\ 4;\ 4\ 5\ 6\ 5;\ 6\ 6\ 6\ 5;\ 2\ 1\ 4\ 7]$
If A is different, (check it by typing A) re-enter A as above to redefine it.

5.1 Examples on how to enter Matrix operations in MATLAB

- $A + B$ by typing $A + B$
- $5C$ by typing $5 * C$
- A^2 by typing $A\hat{}\ 2$
- $A - 3B$ by typing $A - 3 * B$

5.2 Finding power of A and polynomials of A

From now on if in any place in the LABS you are asked to find A^2 You need to enter $A\ \hat{}\ \ 2$
For example if you are asked to find $D = 3A + 2A^2 - A^3 + A^5$ you need to type (enter) in MATLAB as:
$D = 3 * A + 2 * A\hat{}\ 2 - A\hat{}\ 3 + A\hat{}\ 5$.

6 Triangular Matrices

Explore what happens if we add, subtract or multiply triangular matrices? Do we get a Triangular matrix or something else?
Create a 5 by 5 matrix by typing:

U= round(10 * rand(5)).

Similarly create 5 by 5 matrices B and C by typing

V= round(10 * rand(5))
W= round(10 * rand(5))

Type: $L = tril(U)$ to create a lower triangular matrix from U.
Type: $K = tril(V)$ to create another lower triangular matrix from V.
Type: $J = triu(V)$ to create an upper triangular matrix from V.
Now find the following:

- $L - K$
- $3L + 5K$ (Note: you need to type 3*L+5*K)
- LK
- KL
- K^3
- $J + K$
- $5J$
- J^2

Answer the following questions:
a.) Explain: What type of matrix are you getting? Is it lower triangular , upper triangular, or other type that you know?
b.) Is it possible that "the sum of two lower triangular matrices be non-lower triangular matrix" ? Explain.
c.) What do you think about "the product of scalar(number) with a lower triangular matrices should it be a lower triangular matrix" ? Why? Explain.
d.) What do you think about multiplying a lower triangular matrix by a lower triangular matrix will the result be a lower triangular matrix? Explain.
e.) Generalize your findings and extend them to sum, difference, product, and scalar product of upper triangula matrices.For example:
1. Sum of two upper triangula matrices is .....
2. Product of two upper triangula matrices is .....

7 Diagonal Matrices

Diagonal Matrices If $A = (a_{ij})$ is a square matrix, then the entries a_{ii} are called **diagonal entries.** A square matrix is called **diagonal** if all non-diagonal entries are zeros.

Explore what happens if we add, subtract or multiply diagonal matrices. A and B are the same matrices in previous sections (section 5.)

Type D=diag(diag(A)) to create a diagonal matrix from A.

Type E=diag(diag(B)) to create another diagonal matrix from B.

Find the following:

- a) D+E
- b) D-E
- c) DE
- d) ED

Answer the following questions:

- 1.) Explain what type of matrix are you getting?
- 2.) Can you make a statement to generalize this fact?
- 3.) Is it possible to get a non diagonal matrix from adding or multiplying diagonal matrices?
- 4.) Can we obtain a diagonal matrix by multiplying two non-diagonal matrices? Give an example
- 5.) Can we obtain a diagonal matrix by adding two non-diagonal matrices? Give an example

8 Symmetric and skew symmetric matrices

Symmetric A matrix M is called **symmetric** if it is equal to its transpose, that is $M = M'$.

Skew symmetric A matrix M is called **skew symmetric** if it is equal to its transpose, that is $M = -M'$.

Example: Enter the following matrix in MATLAB .

Type: M = [1 1 2 5; 1 7 3 -4; 2 3 8 1; 5 -4 1 9]

Type M'

see if $M = M'$ or $M = -M'$

Example: Enter the following matrix in MATLAB :

M = [0 1 -2 5; -1 0 3 -4; 2 -3 0 6; -5 4 -6 0]

Type M'

see if $M = -M'$

8.1 You can create a symmetric matrix from a given square matrix:

Type $S = A + A'$

to get a symmetric matrix.

Type $T = B + B'$

to get a symmetric matrix.

Type $R = A - A'$

to get a skew symmetric matrix.

8.2 Explore what happens if you add, subtract or multiply symmetric matrices?

NOTE : To enter transpose of a matrix A in MATLAB you need to type A'.

Recall that a matrix A is called symmetric if $A = A'$.

Find the following

- a.) S+T
- b.) S-T
- c.) ST
- d.) TS

Answer the following questions:

a.) Which one of these matrices are symmetric?

b.) What type of matrix will we get if we add (multiply) two symmetric matrices?

c.) Can we get symmetric matrices by adding two non-symmetric matrices?

9 Solve the linear system:

Reading Materials:

There are several ways solving the linear system Solving $AX = b$, we will examine three, you may learn these later in your 22A Class:

9.1 Using the function "rref".

If A is a rectangular matrix and you want to find the general solution of $AX = b$, first enter the augmented matrix of the system by typing $C = [A \;\; b]$, then type $rref(C)$. (You can do these together by typing the Shortcut: rref ([A b])

Example :

To Solve the linear system:

$$\begin{array}{llll} 2x_1 & +4x_2 & -2x_3 & = 0 \\ 3x_1 & +5x_2 & & = 1 \end{array}$$

First we need to enter the augmented matrix

$$M = \left[\begin{array}{cccc} 2 & 4 & -2 & 0 \\ 3 & 5 & 0 & 1 \end{array}\right]$$

then find "rref" form by typing *rref(M)*.

You will see

$$\left[\begin{array}{cccc} 1 & 0 & 5 & 2 \\ 0 & 1 & -3 & -1 \end{array}\right].$$

The corresponding system of equations is:

$$\begin{array}{llll} x_1 & & +5x_3 & = 2 \\ & x_2 & -3x_3 & = -1 \end{array}$$

As you see x_1 and x_2 are leading variables and x_3 is non-leading (free) variable. Now, using the parameter t to represent the $non-leading$ variable x_3, we have the general solution:

$$\begin{array}{llll} x_1 & = & 2- & 5t, \\ x_2 & = & -1+ & 3t, \\ x_3 & = & & t \end{array}$$

If you type X = [2 4 -2; 3 5 0] \ [0; 1]
MATLAB will give you only particular solution.

9.2 Using MATLAB's command X=A \ b to solve a linear system:

In Previous subsection you learned that a system can be solved by "rref" or $A\backslash B$.
Now we like to see how MATLAB responds when we try to solve an inconsistant system using $A\backslash B$..
To solve the following linear system

$$\begin{array}{llll} 2x_1 & +4x_2 & -2x_3 & = 0 \\ 3x_1 & +5x_2 & & = 1 \\ 4x_1 & +8x_2 & -4x_3 & = 3 \end{array}$$

Enter the augmented matrix for this linear system:
Type AG= [2 4 -2 0; 3 5 0 1; 4 8 -4 3]
Type rref(AG) to get **Row Reduced Echelon Form** of the augmented matrix. You should get the following matrix:

$$\begin{array}{cccc} 1 & 0 & 5 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 1 \end{array}$$

This the correspondes to the following system of equations,

$$\begin{array}{llll} x_1 & & +5x_3 & = 0 \\ & x_2 & -3x_3 & = 0 \\ 0 & +0 & +0 & = 1 \end{array}$$

Note that this is an inconsistent system. (Type a % and Explain Why this is an inconsistent system.)
Now type the coefficient matrix:
$AC = [2\ 4\ -2; 3\ 5\ 0; 4\ 8\ -4]$
and the constant matrix as $b = [0\ 1\ 3]'$
then use MATLAB's command
Type $X = AC\backslash b$
How do you Interpret MATLAB's output?
Is it confirming your findings about this linear system by "rref"?

Example:

Enter
Type: $A = \begin{bmatrix} 1 & -1 & -2 \\ 2 & 1 & 3 \\ 2 & 3 & 0 \end{bmatrix}$.
and
Type: B=[3 6 7]'.
To solve this system,
Type X = A\ B
Check the answer Using *rref* Method in section **9.1**
This is the end of the LAB 2.