MATH 22AL
Lab # 8

1 Objectives

In this LAB you will explore the following topics using MATLAB.

- Properties of determinant function
- The effects of row operations on the value of the determinant of a matrix.
- Matrix operations and determinats .
- Determinant of special matrices.

2 What to turn in for this lab

Please save and submit your MATLAB session.
Important: Do not Change the name of the Variables that you supposed to type in MATLAB, type as you are asked.

3 Recording and submitting your work

The following steps will help you to record your work and save and submit it successfully.

- Open a terminal window.
  - In Computer LAB (2118 MSB) click on terminal Icon at the bottom of the screen
  - In Windows OS, Use Putty
  - In MAC OS, Use terminal window of MAC.
- Start a MATLAB Session that is :
  - Type "textmatlab" Press Enter
- Enter your information that is :
  - Type "diary LAB8.text"
  - Type "% First Name:" then enter your first name
  - Type "% Last Name:" then enter your Last name
  - Type "% Date:" then enter the date
  - Type "% Username:" then enter your Username for 22AL account
- Do the LAB that is :
  - Follow the instruction of the LAB.
  - Type needed command in MATLAB.
  - All commands must be typed in front of MATLAB Command " >> "..
- Close MATLAB session Properly that is :
  - When you are done or if you want to stop and continue later do the following:
  - Type "save" Press Enter
  - Type "diary off" Press Enter
  - Type "exit" Press Enter
- Edit Your Work before submitting it that is :
  - Use pico or editor of your choice to clean up the file you want to submit:
  - in command line of pine type "pico LAB6.text"
  - Delete the errors or insert missed items.
  - Save using "ˆ o= control key then o"
  - Exit using "ˆ x= control key then x"
- Send your LAB that is :
  - Type "ssh point" : Press enter
  - Type submitm22al LAB8.text

4 Background, Reading Part : Introduction to Determinants

4.1 Introduction to Determinants

Determinant of a 2X2 square matrix

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

is defined as

$$\det(A) = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

Example 1:

Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

The determinant of A is

$$\det(A) = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = (1)(4) - (2)(3) = -2$$

In general **the determinant of a square matrix** A is a unique number (Scalar) associated with that square matrix A . There are various ways to define the determinant of a square matrix A. And there are various ways to compute $det(A)$. One way to compute it is from the entries of the matrix by a specific arithmetic expression. There are other ways to determine $det(A)$.

The determinant of A provides important information about the matrix of coefficients of a system of linear equations $AX = B$. For example, when the determinant is nonzero the system has a unique solution. If we consider A as a linear transformation of a vector space and A has real entries, a geometric interpretation can be given to the value of the determinant of A. For example, the absolute value of the determinant gives the scale factor by which area or volume is multiplied.

The determinant of a 3×3 matrix A is given by

$$\det(A) = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = aei - afh + bfg - bdi + cdh - ceg$$

Using MATLAB you can find $det(A)$ by typing det(A).

Example 2: Given A=[1 2 ; 3 4] by typing DA = det(A) in MATLAB, you will see DA= -2 which is the number (1)(4) - (2)(3).

Background, Reading Part :

4.2 How to find Determinant of an $n \times n$ matrix?

Determinant of 2×2 and 3×3 matrices are given as:

$$\det(A) = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$\det(A) = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = aei - afh + bfg - bdi + cdh - ceg$$

For an $n \times n$ matrix A, one way to define $det(A)$ is as follow:
I

$$det(A) = |A| = \sum (\pm) a_{1,j_1} a_{1,j_2} a_{1,j_3} \cdots a_{1,j_n}$$

Where summation ranges over all permutations $J_1 J_2 J_3 \cdots j_n$ of the set $\{1, 2, 3, \cdots n\}$. The sign is taken as + or - according to weather the permutation is even or odd.

II Another way to find $det(A)$ is cofactor expansion .

IIII One other method is using row operations to reduce A to a triangular matrix and use the fact that determinant of a triangular matrix A is the product of its diagonal entries.

For more information you may read your text book or the following or other web resources.

Start Typing in MATLAB

5 Explore what happens to the determinant of a matrix A when you perform row operation on A.

5.1 What happens to $det(A)$ when two rows of A are interchanged?

Example 3: Type :

$$B = [1\ 2\ 3; 6\ 7\ 8; 0\ 1\ 1]$$

to enter a 3×3 matrix B.

Interchange second and third rows of the matrix B by typing :
Type :

$$RB = B$$

Type :

$$RB(2,:) = B(3,:)$$

Type :

$$RB(3,:) = B(2,:)$$

You may do all in one row as:
Type :

$$RB = B; RB(2,:) = B(3,:); RB(3,:) = B(2,:)$$

Now type the following to see what happened:
Type :

$$B$$

Type :

$$RB$$

To find out the determinants for RB and B tyoe:
Type :

$$DB1 = det(B)$$

Type :

$$DB2 = det(RB)$$

Explain the relation between $det(B)$ and $det(RB)$ by typing % and entering your statement.

5.2 What happens to $det(A)$ when two rows of A are interchanged?

Example 4:
Now lets look at a new matrix:
Create a 5 by 5 matrix, C by typing:
Type :

$$\mathbf{C = [1\ 2\ \ -1\ 3\ 4; -1\ 0\ 1\ 2\ 1; 10\ 3\ 2\ 1\ 1; -1\ 0\ 2\ 3\ 2; 1\ 1\ 1\ 2\ \ -1]}.$$

Find $det(C)$ by typing:
Type :

$$DC = det(C)$$

Note: You can interchange i^{th} and j^{th} rows of C by typing (i and j must be defined (entered in MATLAB) first):

$$RC = C; RC(i,:) = C(j,:); RC(j,:) = C(i,:); RC$$

For example to interchange the second row and third row of C,
Type :

$$i = 2; j = 3; R23C = C; R23C(i,:) = C(j,:); R23C(j,:) = C(i,:); R23C$$

Type :

$$DR23C = det(R23C)$$
$$DC23 = det(C)$$

Do the following Exercises:

1. Interchange the first and third rows of C and call the resulting matrix R13C. (You need to use the procedure above)

   - Find both determinant of C and R13C by typing Type :

   $$DR13C = det(R13C)$$
   $$DC13 = det(C)$$

   - Explain the relation between the two determinants of R13C and C.

2. Interchange the second and fifth rows of C and call the new matrix R25C, then interchange first and second row of R25C call the resulting matrix R125C.

   Find det(R125C).

   Type :

   $$DR125C = det(R125C)$$
   $$DC125 = det(C)$$

   Compare the determinants of C and R125C. Explain your observation (by typing %).

3. Interchange first and second rows of $R125C$ and call the new matrix R12RC.

   Find det(R12RC).

   Type :

   $$DR12C = det(R12RC)$$
   $$DC12 = det(C)$$

   Compare the determinants of C and R12RC. Explain your observation (by typing %).

   If you need, do more row exchange and make more observations.

4. Make a conjecture about the consequence of row interchange on the value of the determinant of a matrix. Explain your observation (by typing %).

5.3 What happens to $det(D)$ when a row of D is multiplied by a number (Scalar)?

Example 5:
Create a 5 by 5 matrix, C by typing:
Type :

$$\mathbf{D = [1\ 2\ \ -1\ 3\ 4; 1\ 0\ \ -1\ \ -2\ \ -1; 10\ 3\ 2\ 1\ 1; -1\ 0\ 2\ 3\ 2; 1\ 1\ 1\ 2\ \ -1]}.$$

Find $det(D)$ by typing:
Type :

$$\mathbf{D1D = det(D)}.$$

Now we want to observe how determinant changes when one of the rows of the matrix is multiplied by a number.

Note: Multipling the i^{th} row of D by a number (scalar) k in MATLAB can be done as follow. Values of i and k must be defined (entered) first. In the following line we choose $i = 3$ and $k = 45$.
Type :

$$\mathbf{i = 3; k = 45; R1D = D; R1D(i, :) = k \star D(i, :); R1D}.$$

Type :

$$DR1D = det(R1D)$$
$$DD1 = det(D)$$
$$DK1 = k * det(D)$$

Compare the determinants of D and $R1D$. Explain your observation (by typing %).

Do the following Exercises:

1. Multiply the first row of D by 3 and call the new matrix R2D. (Use the procedure in Example 5 in previous section)

   Type :

$$\mathbf{i = 1; k = 3; R2D = D; R2D(i, :) = k \star D(i, :); R2D}.$$

   Type :

$$DR2D = det(R2D)$$
$$DD2 = det(D)$$
$$DK2 = k * det(D)$$

   Compare the determinants of D and $R2D$. Explain your observation (by typing %).

2. Multiply the third row of D by -5 and call the new matrix $R3D$.

   Type :

$$\mathbf{i = 3; k = -5; R3D = D; R3D(i, :) = k \star D(i, :); R3D}.$$

   Type :

$$DR3D = det(R3D)$$
$$DD3 = det(D)$$
$$DK3 = k * det(D)$$

   Compare the determinants of D and $R3D$. Explain your observation (by typing %).

3. Repeat this experiment using different rows of D .That is multiply the a row of D by an scalar and call the new matrix $R4D$. Choose your own row, and scalar.

   Type :

$$\mathbf{i = ??; k = ??; R4D = D; R4D(i, :) = k \star D(i, :); R4D}.$$

   Type :

$$DR4D = det(R4D)$$
$$DD4 = det(D)$$
$$DK4 = k * det(D)$$

   Compare the determinants of D and $R4D$. Explain your observation (by typing %).

4. Can you generalize this fact? Make a conjecture about the effect of multiplying a row of a matrix by an scalar, on the value of the determinant of a matrix.

5.4 ADDING A MULTIPLE OF THE i^{th} ROW TO THE j^{th} row.

Example 6:
Create a 5 by 5 matrix, E by typing:
Type :

$$\mathbf{E} = [\mathbf{1\ 2\ -1\ 3\ 4; 1\ 0\ -1\ -2\ -1; 8\ 3\ 2\ 1\ 1; 1\ 0\ -2\ -3\ -2; 1\ 1\ 1\ 2\ -1}].$$

Find $det(E)$ by typing:
Type :

$$DE = det(E)$$

Note: Adding a multiple of i^{th} row of E to the j^{th} row in MATLAB can be done as follow. Values of i, j and k must be defined (entered) first. In the following line we choose $i = 3, j = 5$ and $k = 2$.
Type :

$$\mathbf{i} = \mathbf{3}; \mathbf{j} = \mathbf{5}; \mathbf{k} = \mathbf{2}; \mathbf{R1E} = \mathbf{E}; \mathbf{R1E}(\mathbf{j}, :) = \mathbf{k} \star \mathbf{E}(\mathbf{i}, :) + \mathbf{E}(\mathbf{j}, :); \mathbf{R1E}.$$

Type :

$$DR1E = det(R1E)$$
$$DE1 = det(E)$$

Compare the determinants of E and $R1E$. Explain your observation (by typing %).

Do the following Exercises:

1. Multiply the first row of E by 3 and add it to the second row of E call the new matrix R2E. Type :

$$\mathbf{i = 1; j = 2; k = 3; R2E = E; R2E(j,:) = k \star E(i,:) + E(j,:); R2E.}$$

   Type :

$$DR2E = det(R2E)$$
$$DE2 = det(E)$$

   Compare the determinants of E and $R2E$. Explain your observation (by typing %).

2. Multiply the second row of E by 16 and add it to the fifth row of E call the new matrix R3E. Type :

$$\mathbf{i = 2; j = 5; k = 16; R3E = E; R3E(j,:) = k \star E(i,:) + E(j,:); R3E.}$$

   Type :

$$DR3E = det(R3E)$$
$$DE3 = det(E)$$

   Compare the determinants of E and $R3E$. Explain your observation (by typing %).

3. Repeat this experiment using different rows of E. That is multiply the a row of E by an scalar and add it to another row of E, call the new matrix $R4E$. Choose your own rows, and scalar.

   Type :

$$\mathbf{i =??; j =??; k =??; R4E = E; R4E(j,:) = k \star E(i,:) + E(j,:); R4E.}$$

   Type :

$$DR4E = det(R4E)$$
$$DE4 = det(E)$$

   Compare the determinants of E and $R4E$. Explain your observation (by typing %).

4. Can you generalize this fact? Make a conjecture about the effect adding a scalar multiple of a row to another row of a matrix, on the value of the determinant of a matrix.

6 Explore what is the determinant of FG, and H^n.

Create a 5 by 5 matrix, F by typing:
Type :

$$\mathbf{F} = [\mathbf{1}\ \mathbf{2}\ -\mathbf{1}\ \mathbf{3}\ \mathbf{4}; \mathbf{1}\ \mathbf{0}\ -\mathbf{1}\ -\mathbf{2}\ -\mathbf{1}; \mathbf{8}\ \mathbf{3}\ \mathbf{2}\ \mathbf{1}\ \mathbf{1}; \mathbf{1}\ \mathbf{0}\ -\mathbf{2}\ -\mathbf{3}\ -\mathbf{2}; \mathbf{1}\ \mathbf{1}\ \mathbf{1}\ \mathbf{2}\ -\mathbf{1}].$$

Similarly create 5 by 5 matrices G and H, by typing
Type :

$$\mathbf{G} = [\mathbf{1}\ \mathbf{1}\ \mathbf{1}\ \mathbf{2}\ \mathbf{0}; \mathbf{0}\ \mathbf{4}\ \mathbf{1}\ \mathbf{0}\ \mathbf{1}; \mathbf{1}\ \mathbf{2}\ \mathbf{1}\ \mathbf{1}\ \mathbf{0}; \mathbf{0}\ \mathbf{1}\ \mathbf{1}\ \mathbf{1}\ \mathbf{0}; \mathbf{0}\ \mathbf{0}\ \mathbf{1}\ \mathbf{0}\ \mathbf{0}]$$

Type :

$$\mathbf{H} = [\mathbf{2}\ \mathbf{1}\ \mathbf{1}\ \mathbf{2}\ \mathbf{0}; \mathbf{0}\ \mathbf{4}\ \mathbf{1}\ \mathbf{0}\ \mathbf{1}; \mathbf{1}\ \mathbf{2}\ \mathbf{1}\ \mathbf{1}\ \mathbf{0}; \mathbf{0}\ \mathbf{1}\ \mathbf{1}\ \mathbf{1}\ \mathbf{0}; \mathbf{0}\ \mathbf{0}\ \mathbf{1}\ \mathbf{0}\ \mathbf{0}]$$

a.) Find the following determinants:
Type :

$$\det(\mathbf{F})$$
$$\det(\mathbf{G})$$
$$\det(\mathbf{H})$$
$$\det(\mathbf{F} \star \mathbf{G})$$
$$\det(\mathbf{G} \star \mathbf{F})$$
$$\det(\mathbf{F} \star \mathbf{H})$$
$$\det(\mathbf{H} \star \mathbf{F})$$
$$\det(\mathbf{G} \star \mathbf{H})$$
$$\det(\mathbf{H} \star \mathbf{G})$$
$$\det(\mathbf{H} \star \mathbf{G} \star \mathbf{G})$$
$$\det(\mathbf{H} \star \mathbf{G} \star \mathbf{F})$$
$$\det(\mathbf{H} \star \mathbf{H} \star \mathbf{H})$$
$$\det(\mathbf{G} \star \mathbf{G} \star \mathbf{H})$$
$$\det(\mathbf{F} \star \mathbf{F} \star \mathbf{F})$$
$$\det(\mathbf{G} \star \mathbf{G})$$
$$\det(\mathbf{G} \star \mathbf{G} \star \mathbf{G})$$
$$\det(\mathbf{G} \star \mathbf{G} \star \mathbf{G} \star \mathbf{G})$$

Observation what is going on regarding determinant of product of two matrices.

a1.) Make a conjecture about the relation between $det(AB)$, and $det(BA)$. Type your answer after %.

a2.) Make a conjecture about the relation between $det(AB), det(B)$, and $det(A)$. Type your answer after %.

a3.) Make a conjecture about the relation between $det(A^n)$, and $det(A)$. Type your answer after %.

7 Explore what is the determinant of kA.

Create a 5 by 5 matrix, F by typing:
Type :

$$\mathbf{F = [1\ 2\ -1\ 3\ 4; 1\ 0\ -1\ -2\ -1; 8\ 3\ 2\ 1\ 1; 1\ 0\ -2\ -3\ -2; 1\ 1\ 1\ 2\ -1]}.$$

a.) Find the following determinants:
Type :

$$\det(\mathbf{F})$$
$$\det(\mathbf{2 \star F})$$
$$\det(\mathbf{-3 \star F})$$
$$\det(\mathbf{10 \star F})$$

b.) Make a conjecture about the relation between $det(A)$, and $det(kA)$ where k is a number (scalar). Type your answer after %.

8 Explore what is the determinant of AB, and A^{-1}.

Create a 5 by 5 matrix, F by typing:
Type :

$$\mathbf{F = [1\ 2\ \ -1\ 3\ 4; 1\ 0\ \ -1\ \ -2\ \ -1; 8\ 3\ 2\ 1\ 1; 1\ 0\ \ -2\ \ -3\ \ -2; 1\ 1\ 1\ 2\ \ -1]}.$$

Create a 3 by 3 matrix, FF by typing:
Type :

$$\mathbf{FF = [1\ 2\ \ -1; 1\ 0\ \ -1; 1\ \ -1\ 1]}.$$

a.) Find the following determinants:
Type :

$$\det(\mathbf{F})$$
$$\det(\mathbf{inv}(\mathbf{F}))$$
$$\det(\mathbf{inv}(\mathbf{inv}(\mathbf{F})))$$
$$\det(\mathbf{FF})$$
$$\det(\mathbf{inv}(\mathbf{FF}))$$
$$\det(\mathbf{inv}(\mathbf{inv}(\mathbf{FF})))$$

b.) Make a conjecture about the relation between $det(A)$, and $det(A^{-1})$. Type your answer after %.

9 Explore determinant of $A + B$, and $A - B$.

Type :

$$\mathbf{HH} = [\mathbf{1\ 2\ \ -1; 1\ 0\ \ -1; 1\ \ -1\ 1}].$$

Type :

$$\mathbf{GG} = [\mathbf{1\ 2\ \ -1; 2\ 0\ \ -2; 1\ \ -1\ 1}].$$

a.) Find the following determinants:
Type :

$$\det(\mathbf{HH})$$
$$\det(\mathbf{GG})$$
$$\det(\mathbf{HH} + \mathbf{GG}))$$
$$\det(\mathbf{HH} - \mathbf{GG})$$

There is no known simple general relation in these cases. There are some known relations when we restrict A and B. You may look to this on web or in your book for more information.

10 Explore what is the determinant of A, and A'.

Type :

$$\mathbf{K=[1\ 2\ \ -1;1\ 0\ \ -1;1\ \ -1\ 1]}.$$

Type :

$$\mathbf{L=[1\ 2\ \ -1;2\ 0\ \ -2;1\ \ -1\ 1]}.$$

a.) Find the following determinants:
Type :

$$\det(\mathbf{K})$$
$$\det(\mathbf{L})$$
$$\det(\mathbf{K'})$$
$$\det(\mathbf{L'})$$

As you may observe $det(A) = det(A')$. This implies that any properties that we stated about how row operations and determinant is related can be extend to the columns of a matrix.