MAT 16C: Practice Final
Monday, July 25th

Write legibly and neatly. Show all your work and simplify answers for full credit.
1. (a) Find the general solution of the following differential equation:

\[ \frac{dy}{dt} = 3 \log(t + 1) \frac{t}{t + 1} \]

Be sure to solve for \( y \).

(b) Find the particular solution given that \( y = 4 \) when \( t = 0 \).
2. Find the general solution to the differential equation:

\[ y' + 2xy = x. \]
3. TRUE / FALSE. Circle the choice that you think is correct. Justification is not necessary, and will not be taken into account when grading.

(a) TRUE / FALSE - The differential equation
\[ y' = y^2 x - y^2 e^x \]

is separable.

(b) TRUE / FALSE - The equation \( y = \sin(x^2) \) is a solution to the differential equation \( y'' = 4x^2y \).
4. Use the method of Lagrange Multipliers to find the maximum of the function:

\[ f(x, y) = xyz, \]

subject to the constraint:

\[ x + y + z - 6 = 0. \]

You will receive no credit if you do not use the method of Lagrange Multipliers.
5. TRUE / FALSE. Circle the choice that you think is correct. Justification is not necessary, and will not be taken into account when grading.

(a) TRUE / FALSE - The equation $z^2 + x^2 = 1 + y^2$ is the equation of an elliptic cone.

(b) TRUE / FALSE - If $f_{xx}(a, b) = 4$, $f_{yy}(a, b) = 6$, and $f_{xy}(a, b) = -3$, then $(a, b)$ is a relative maxima.

(c) TRUE / FALSE - You can obtain an ellipse by taking a trace of an hyperboloid (either type).
6. Evaluate the double integral:

\[ \int_{2}^{3} \int_{e}^{e^x} \frac{x}{y} dy \, dx \]
7. (a) Let $R$ be the region in the first quadrant of the $xy$-plane bounded by $x = y^2$, $x = 4$, and $y = 0$. Sketch the region $R$. This only needs to be a 2D sketch! Please label all lines, functions, and points of intersection.

(b) Fill in the blanks on the following two expressions for the area of $R$. You don’t need to evaluate either of them!

\[ A = \int \int dy\,dx \]

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8. Find the volume above the region $R$ from problem 6 and under the surface:

\[ e^{x^{3/2}}. \]

HINT: One order of integration is much easier than the other!
9. Determine whether or not each of the following sequences converges or diverges. If they converge, what do they converge to? You do not need to justify your answers.

(a) \( a_n = \frac{(n + 1)!}{n!} \)

(b) \( b_n = \frac{1}{n^2} \)

(c) \( c_n = \frac{\sin(n)}{e^n} \)

(d) \( d_n = n^{-n} e^n \)
10. Determine whether or not each of the following series converges or diverges. You must justify your answer! This means both using a given test correctly and naming the test! **You do not need to find the sum of the series.**

(a) \[ \sum_{n=2}^{\infty} \frac{n^4 + 6}{n^5} \]

(b) \[ \sum_{n=1}^{\infty} \frac{e^n}{\pi^{n+1}} \]

(c) \[ \sum_{n=0}^{\infty} \frac{(2n)!}{n!n!} \]

(d) \[ \sum_{n=0}^{\infty} (n+1)^{-\sqrt{7}} \]
11. Provide an example of a series, ∑\( a_n \), such that ∑\( a_n \) converges but ∑\( \sqrt{a_n} \) diverges. If you write down both series, make sure that I can tell which series is which!
12. (a) Provide an example of a geometric series.

(b) What is the sum of your series? (Show work).
13. Find the radius of convergence of the following power series. You must fully justify your answers.

(a) \[ \sum_{n=0}^{\infty} \frac{n}{n+1} x^n \]

(b) \[ \sum_{n=0}^{\infty} \frac{x^{2n+1}}{n!} \]
14. Estimate the integral of:

\[ \int_{1/4}^{3/4} \frac{1}{2x^5 + x} \, dx \]

Using the first three terms of an expansion for the integrand. You may find it helpful to know that:

\[ \frac{1}{x+1} = 1 - x + x^2 - x^3 + x^4 - x^5 + \cdots \]

(note: this is going to end up with some stuff you’ll need a calculator for... you won’t need one on the exam.)