MAT 16C: Quiz 1 - Practice
Friday, June 24

Write legibly and neatly. Show all your work for full credit.

1. (10 points)

Use an integrating factor to find the general solution to the differential equation:

\[
\frac{y'}{3x^2} + y = 1
\]

We begin by writing the equation in standard form:

\[
y' + 3x^2y = 3x^2
\]

So we see that \(P(x) = 3x^2\) and \(Q(x) = 3x^2\). We can then calculate the integrating factor, \(u(x)\):

\[
u(x) = e^{\int P(x) \, dx} = e^{\int 3x^2 \, dx} = e^{x^3}
\]

We can now calculate the solution using the formula:

\[
y(x) = \frac{1}{u(x)} \int Q(x) u(x) \, dx
\]

\[
= \frac{1}{e^{x^3}} \int 3x^2 e^{x^3} \, dx
\]

\[
= e^{-x^3} \left( e^{x^3} + C \right)
\]

\[
= 1 + Ce^{-x^3}
\]
2. **(5 points)** A diameter of a sphere has the endpoints \((4, 1, -2)\) and \((4, -5, 6)\).

(a) Find the center of the sphere.

The center of the sphere will be at the midpoint of the diameter. So to find the center, we can just find the midpoint between the two given points:

\[
\center = \left( \frac{4 + 4}{2}, \frac{1 + (-5)}{2}, \frac{-2 + 6}{2} \right) = (4, -2, 2)
\]

So the center of the sphere is at the point \((4, -2, 2)\).

(b) Find the radius of the sphere.

To find the radius of the sphere, we just find the distance between the center of the sphere and one of the points on its diameter:

\[
r = \sqrt{(4 - 4)^2 + (1 - (-2))^2 + (-2 - 2)^2} = \sqrt{0^2 + 3^2 + (-4)^2} = \sqrt{0 + 9 + 16} = 5
\]

(c) Write the equation of the sphere in its standard form.

\[
(x - 4)^2 + (y + 2)^2 + (z - 2)^2 = 25
\]
3. **(10 points)** Let $S$ be the surface given by the equation:

$$x = 4z^2 - 8y^2$$

a. (2pts) Find the xy-trace and classify it as a parabola, hyperbola, or ellipse.

We obtain the xy-trace by setting $z = 0$. This gives the equation:

$$x = -8y^2$$

Which is a parabola.

b. (2pts) Find the xz-trace and classify it as a parabola, hyperbola, or ellipse.

We obtain the xz-trace by setting $y = 0$. This gives the equation:

$$x = 4z^2$$

Which is a parabola.

c. (2pts) Find the intersection of $S$ and the plane $x = 4$. Classify it as a parabola, hyperbola, or ellipse.

We obtain the intersection of $S$ and the plane $x = 4$ by letting $x = 4$. This gives the equation:

$$4 = 4z^2 - 8y^2$$
$$1 = z^2 - 2y^2$$

Which is the equation of an hyperbola.

d. (4pts) Classify the surface as one of the six types of quadratic surfaces we learned about in class.

Hyperbolic Paraboloid.