Write legibly and neatly. Show all your work for full credit. This practice test is longer than the real exam will be.

1. Let \( f(x) = x + x^2 \) on the interval \([0, 3]\).
   (a) Find a formula for the Riemann sum obtained by dividing the given interval into \( n \) equal subintervals and use the right-hand endpoint for each \( c_k \).
   (b) Take the limit of your formula from part (a) to compute the area under the curve \( y = f(x) \) on the given interval. It may be useful for you to use the formula:
   \[
   \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}
   \]
   (c) Compute the integral using the fundamental theorem of calculus and show that these agree.

2. The inequality \( \sec x \geq 1 + \frac{x^2}{2} \) holds on \((−\pi/2, \pi/2)\). Use this inequality to find an upper bound for the value of \( \int_{0}^{1} \sec x \, dx \).

3. Find the derivative \( \frac{dy}{dx} \) of the function:
   \[
y = x \int_{x}^{x^2} \sin(t^3) \, dt
   \]

4. Let \( f \) be any continuous function on \([0, a]\). Compute:
   \[
   \int_{0}^{a} \frac{f(x)}{f(x) + f(a-x)}
   \]
   **Hint:** Let \( I = \int_{0}^{a} \frac{f(x)}{f(x) + f(a-x)} \). Now make the substitution \( u = a - x \). You’ll get a new integral. Since it’s a definite integral, after you’ve completely done the substitution, you can replace the symbol ‘\( u \)’ with the symbol ‘\( x \)’. Now add the second integral to the first integral, and you’ll get \( 2I = \) some other integral which should simplify nicely.

5. Find the total area between the graph of the function \( y = x\sqrt{4-x^2} \) and the x-axis on the interval \([-2, 2]\).

10. (a) Compute the definite integral:
\[ \int_{0}^{1} \theta \cos \pi \theta \, d\theta. \]
(b) Compute the indefinite integral:
\[ \int e^{2x} \cos 3x \, dx. \]
11. Compute the indefinite integral:
\[ \int \sec^3 x \tan^3 x \, dx. \]
12. Compute the average value of the function:
\[ f(x) = \frac{3x + 1}{x^2 - 2x - 15}, \]
from \( x = 0 \) to \( x = 2 \).
13. Compute the indefinite integral:
\[ \int \frac{x^4}{(1 - x^2)^{7/2}} \, dx. \]
Note: You may assume that \(-1 < x < 1\).
14. (a) Use the trapezoidal rule and 6 equal length subintervals to compute an estimate to:
\[ \int_{0}^{3} e^x \, dx \]
You will have to use a calculator to do this (of course, no calculators will be allowed on the real exam!)
(b) Compute a bound for the error.
(c) Compute the exact integral, and find the error between your approximation and the exact integral. Do you satisfy the bound that you found?
(d) Compute a bound for the error had you used Simpson's rule, instead.
15. The intensity \( L(x) \) of light \( x \) feet beneath the surface of the ocean satisfies the differential equation:
\[ \frac{dL}{dx} = -kL. \]
As a diver, you know from experience that diving to 18 feet in the Caribbean Sea cuts the intensity in half. You cannot work without artificial light when the intensity falls below one-sixteenth of the surface value. About how deep can you expect to work without artificial light?
16. Solve the initial value problem:
\[ \sqrt{x} \frac{dy}{dx} = e^{y + \sqrt{x}}, \]
\[ y(0) = 1. \]
17. Let \( \frac{dx}{dt} \) be given by:
\[ \frac{dx}{dt} = \sin x \]
Determine all equilibrium values - note there are infinitely many! Draw a phase line for this differential equation. Make sure to label all equilibrium values, and denote whether they are stable or unstable. Draw arrows indicating how $x$ changes when not at an equilibrium value. Of course, the phase line is infinitely long, so you can’t draw all of it that is relevant. Draw a large enough portion to ensure that you really know what is going on.

18. Does the following integral converge or diverge? If it converges, find its value.

$$
\int_0^1 x \ln x \, dx.
$$

19. Does the following integral converge or diverge? You must justify your answer.

$$
\int_1^\infty \frac{1}{xe^t} \, dx.
$$

20. (a) Find a parametrization for the circle of radius 2, centered at $(2, 2)$.

(b) Compute the arc length of the curve from part (a).

(c) This can also be found using a standard geometric formula. Check that you got the right answer.

21. Eliminate $t$ to find an equation for the curve given by:

$$
x = t^2, \quad y = t^6 - 2t^4, \quad -\infty < t < \infty
$$

Sketch the graph of this curve.

22. Find an equation for the line tangent to the curve:

$$
x = -\sqrt{t + 1} \\
y = \sqrt{3}t
$$

At $t = 3$. What is the concavity at that point?

23. Find the length of the curve:

$$
x = \cos t \\
y = t + \sin t
$$

From $0 \leq t \leq \pi$.

24. Find an equation in cartesian coordinates for the graph given by:

$$
r = 4 \csc \theta
$$

25. Sketch a graph of the polar curve given by:

$$
r = 2 + \cos \theta
$$

26. Find the length of the curve:

$$
r = \frac{6}{1 + \cos \theta}
$$

for $\theta$ in the interval $0 \leq \theta \leq \frac{\pi}{2}$.

27. Book problem 11.5 #2.