MAT 21b: Quiz 7
Thursday, December 4th, 2014

Write legibly and neatly. Show all your work for full credit. You have 20 minutes to complete this quiz.

1. (10 points) Does the following integral converge or diverge? You must justify your answer.

\[ \int_{2}^{\infty} \frac{1}{x^2 \ln x} \, dx \]

It converges. We can see this by using the comparison test.

Since \( \frac{1}{x^2 \ln x} \leq \frac{1}{x^2} \) on \([e, \infty)\),

then since \( \int_{e}^{\infty} \frac{1}{x^2} \, dx \) converges,

so to does \( \int_{2}^{\infty} \frac{1}{x^2 \ln x} \, dx \).

The integral \( \int_{2}^{\infty} \frac{1}{x^2 \ln x} \, dx \) is clearly finite.

Since \( \frac{1}{x^2 \ln x} \) is continuous on \([2, e]\).
2. (10 points)
One week after starting an ant farm, the population is measured to be 1000. One week after that, it's measured at 2000. Assume the ants population is growing according to the exponential growth model. What was their initial population? As always, you must justify your answer.

\[
\frac{dP}{dt} = rP
\]

So \[\int \frac{dP}{P} = \int r \, dt \implies \ln P = rt + C\]

\[P = Ce^{rt}\]

at \(t = 1, \) \(2000 = Ce^{2r}\)

\(t = 1, \) \(1000 = Ce^{r}\)

dividing gives: \(2 = e^{2r}/e^{r} = e^{r}\)

so \(r = \ln 2\)

now \(1000 = Ce^{r} = Ce^{\ln 2} = C2\)

\[\implies C = 500\]

So at \(t = 0, \) \(P = 500e^{r(0)} = \boxed{500} = P\)

also, note that it took 1 week for the population to double — since this is exponential growth, that will always be the case. So you can get to the answer very quickly.
3. (10 points) Let \( \frac{dy}{dt} \) be given by:

\[
\frac{dy}{dt} = y^3 - y
\]

Determine all equilibrium values. Draw a phase line for this differential equation. Make sure to label all equilibrium values, and denote whether they are stable or unstable. Draw arrows indicating how \( y \) changes when not at an equilibrium value.

\[
\frac{dy}{dt} = y(y^2 - 1) = y(y - 1)(y + 1)
\]

Equilibrium values are \( y = 0, 1, -1 \).

\[
(-) -1 (+) 0 (-) 1 (+)
\]