

DIM = 5

A Neighboring polytope not combinatorially equivalent to $C(8,4)$

VALID

-30 25 1 30 -25

INEQUALITIES_SECTION

- (1) $+x_1+x_2+x_3+x_4+x_5 == 1$
- (1) $- 77x_2 - 47x_5 \leq 0$
- (2) $- 47x_2-750x_3 - 77x_5 \leq 0$
- (3) $- 90x_3- 47x_4- 60x_5 \leq 0$
- (4) $- 47x_3-626x_4-750x_5 \leq 0$
- (5) $- 77x_4- 90x_5 \leq 0$
- (6) $- x_2 \leq 0$
- (7) $- 25x_3 - x_5 \leq 0$
- (8) $- x_3 \leq 0$
- (9) $- x_4 \leq 0$
- (10) $- x_2- x_3 - x_5 \leq 0$
- (11) $+ x_3 \leq 1$
- (12) $+ x_2- 29x_3+ x_4+ x_5 \leq 1$
- (13) $+ x_2+ x_3 + x_5 \leq 1$
- (14) $+ 3x_2+ 3x_3- 25x_4- 27x_5 \leq 3$
- (15) $- 2x_2+ 28x_3+ 25x_4+ 28x_5 \leq 28$
- (16) $- 13x_2+ 47x_3+ 47x_4+ 47x_5 \leq 47$
- (17) $+ 47x_2+ 47x_3-626x_4-703x_5 \leq 47$
- (18) $- 13x_2+ 17x_3+ 77x_4+ 77x_5 \leq 77$
- (19) $-124x_2+549x_3+626x_4+626x_5 \leq 626$
- (20) $- 77x_2+643x_3+626x_4+673x_5 \leq 673$

END

strong validity table :

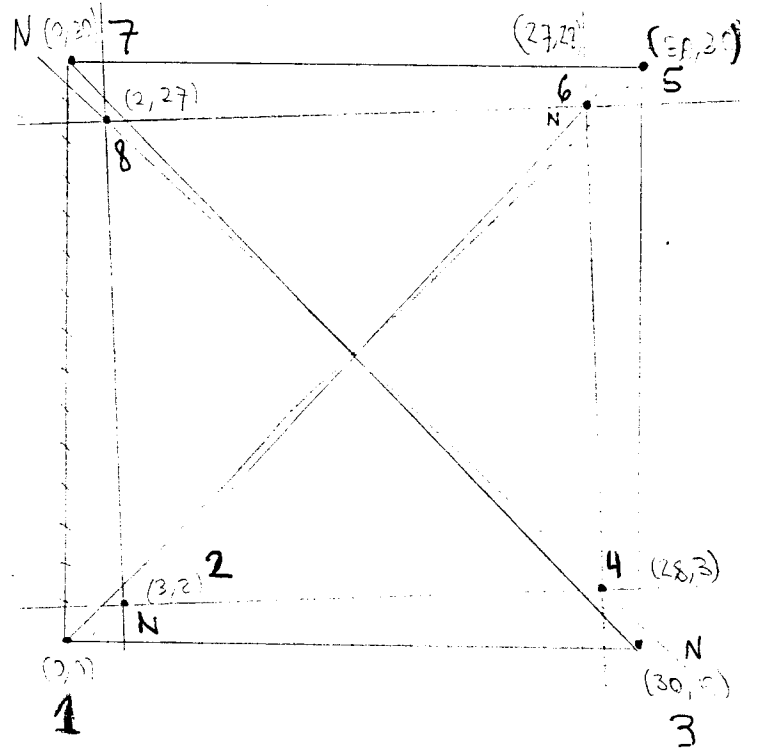
\ P			
\ O			
I \ I			
N \ N	1	6	#
E \ T			
Q \ S			
S \			

1	*.***	4
2	*...*	4
3	**...*	4
4	***..*	4
5	****.	4
6	*...**	4
7	**...*	4
8	**...*	4
9	**...*	4
10	*...*	4
11	...**	4
12	*...*	4
13	*...*	4
14	*...*	4
15	...**	4
16	...***	4
17	...***	4
18	...**	4
19	...**	4
20	...**	4
#	11111 111	
	00000 000	

The edge 28 is contained in the 5 orange facets.

But in $C(8,4)$ an edge can be incident with either 3, 4 or 6 facets, never with 5.

Affine Gale diagram.



Last time, I showed you an Affine Gale Diagram of a neighborly 4-polytope which is NOT the cyclic polytope:

I would like to 'review' all the details of the example:

I provided you with a set of 8 points in the plane. They were (ORDER is important!).

$$(0,0) - \overset{N}{(3,2)} - \overset{N}{(30,0)} - \overset{N}{(28,3)} - \overset{N}{(30,30)} - \overset{N}{(27,28)} - (0,30) - (2,27).$$

NOTE: some are marked negative (N) the rest unmarked. WHERE is the polytope??

1) To read off the faces from the point configuration (see figure in next page) we must find POSITIVE VECTORS (pairs of sets $Z+, Z-$ such that 1) $Z+$ is subset of unmarked points $Z-$ is a subset of marked-negative points and 2) $\text{convexhull}(Z+) \cap \text{convexhull}(Z-)$

If $Z+, Z-$ is a positive vector, then $\{1,2,\dots,8\} - Z+ - Z-$ is the corresponding face for the polytope.

REASON: $(Z+, Z-)$ in the (primal) matroid defined by the vertices of the polytope is a positive COVECTOR. Covectors define faces.

EXAMPLE: The circuit $\{2,6\}, \{4,8\}$ gives the facet $\{1,3,5,7\}$.

The circuit $\{1,4,8\}, \{2\}$ gives us the facet $\{3,5,6,7\}$.

The vector $\{1,5\}, \{2,3,6,7\}$ gives us the face $\{4,8\}$.

The vector $\{1,4,5\}, \{3,6,7\}$ gives us the edge $\{2,8\}$.

In the next page you get a list of facet defining hyperplanes for this polytopes. I got the equations from the coordinates below in part 2

2) The points in the Affine Gale diagram are in fact projections of 8 points in R^3 into the hyperplane $x_3=1$. So originally we had a standard Gale diagram:

$$U := \begin{bmatrix} 0 & -3 & -30 & 28 & 30 & -27 & 0 & 2 \\ 0 & -2 & 0 & 3 & 30 & -28 & -30 & 27 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \end{bmatrix}$$

This is a Gale diagram of the polytope below. The vertices are the columns of the following Matrix (our 4-polytope in question!). It is embedded in R^5 but lies in hyperplane (remember, if a matroid is cyclic, like the vector configuration giving a Gale diagram, then its dual is acyclic).

$$V := \begin{bmatrix} 1 & 0 & -2 & 0 & 0 & 0 & -27 & -30 \\ 0 & 1 & 47 & 0 & 0 & 0 & 673 & 25 \\ 0 & 0 & 30 & 0 & 0 & 0 & 30 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 3 & 0 & 1 & 0 & 28 & 30 \\ 0 & 0 & 77 & 0 & 0 & 1 & 703 & -25 \\ 0 & 0 & 30 & 0 & 0 & 1 & 30 & 0 \end{bmatrix}$$

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$$V := \begin{bmatrix} 1 & 0 & -2 & 0 & 0 & 0 & -27 & -30 \\ 0 & 1 & - & 0 & 0 & 0 & - & 25 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 3 & 0 & 1 & 0 & 28 & 30 \\ 0 & 0 & - & 0 & 0 & 1 & - & -25 \\ 1 & 0 & -2 & 0 & 0 & 0 & -27 & -30 \\ & & 47 & & & & 673 & \\ & & 30 & & & & 30 & \\ & & 77 & & & & 703 & \\ & & 30 & & & & 30 & \end{bmatrix}$$

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A Neighboring polytope not combinatorially equivalent to $C(8,4)$

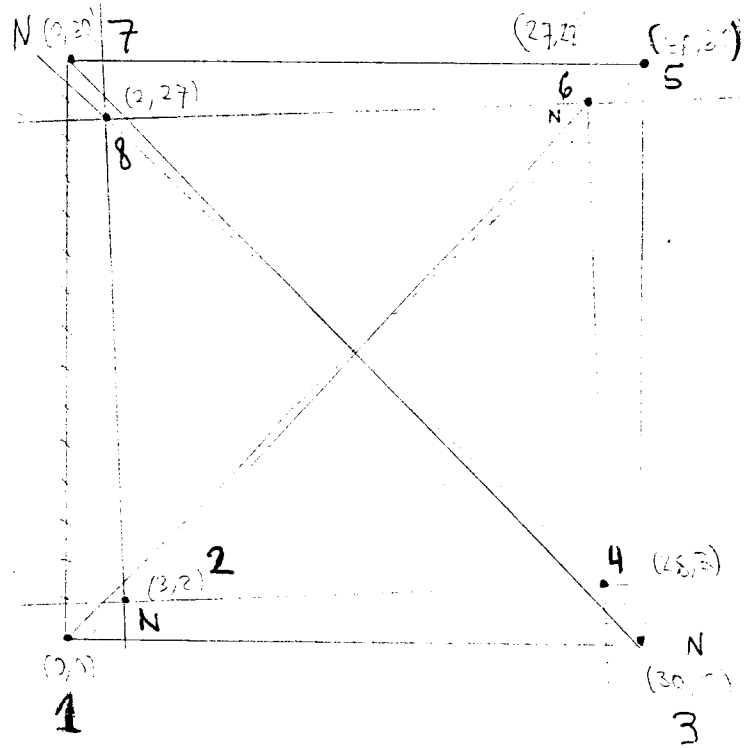
VALID

-30 25 1 30 -25

INEQUALITIES_SECTION

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- (2) - 47x2-750x3 - 77x5 <= 0
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- (6) - x2 <= 0
- (7) - 25x3 - x5 <= 0
- (8) - x3 <= 0
- (9) - x4 <= 0
- (10) - x2- x3 - x5 <= 0
- (11) + x3 <= 1
- (12) + x2- 29x3+ x4+ x5 <= 1
- (13) + x2+ x3 + x5 <= 1
- (14) + 3x2+ 3x3- 25x4- 27x5 <= 3
- (15) - 2x2+ 28x3+ 25x4+ 28x5 <= 28
- (16) - 13x2+ 47x3+ 47x4+ 47x5 <= 47
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Affine Gale diagram.



END

strong validity table :

P	O	I	N	E	Q	S
1	1	6	#			
2	1	6	#			
3	1	6	#			
4	1	6	#			
5	1	6	#			
6	1	6	#			
7	1	6	#			
8	1	6	#			
9	1	6	#			
10	1	6	#			
11	1	6	#			
12	1	6	#			
13	1	6	#			
14	1	6	#			
15	1	6	#			
16	1	6	#			
17	1	6	#			
18	1	6	#			
19	1	6	#			
20	1	6	#			
#	11111	111				
	00000	000				

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N
N
N
N

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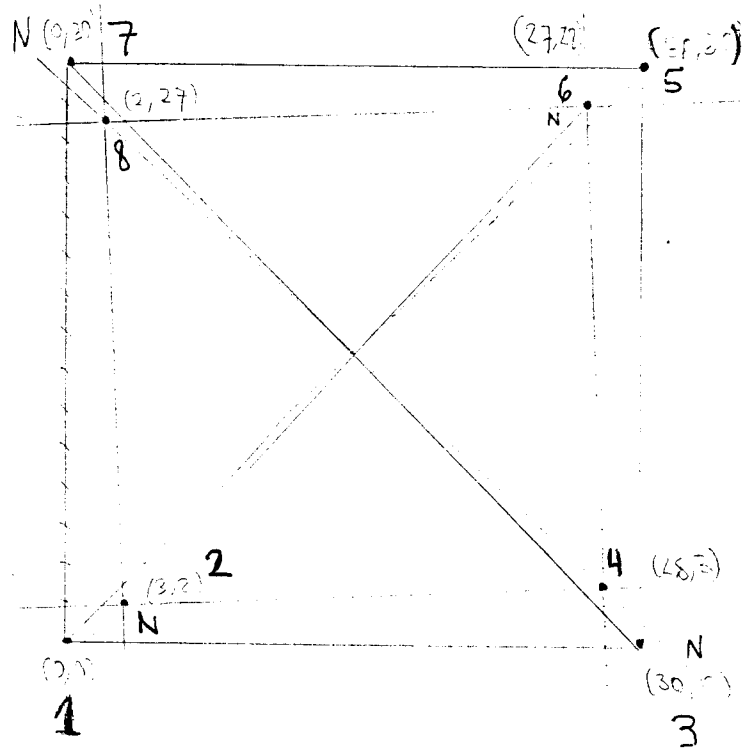
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Affine Gale diagram.



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\ P			
\ O			
I \ I			
N \ N	1	6	#
E \ T			
Q \ S			
S \			

1	*.***	...	4
2	*...*	***	4
3	**...*	***	4
4	***..	.*	4
5	****.	...	4
6	*...*	*..	4
7	**...*	*..	4
8	**...*	*..	4
9	**...*	*..	4
10	*...*	*..	4
11	..**.	**	4
12	*...*	*..	4
13	*...*	*..	4
14	*...*	*..	4
15	..**.	*..	4
16	..***	*..	4
17	..***	*..	4
18*	***	4
19	..**.	**	4
20	..*..	***	4
#	11111	111	
	00000	000	

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