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The Power of Non-Rectilinear Holes*

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Abstract: Four multiconnected-polygon partition problems are shown to be NP-hard.

Introduction

One of the main topics of computational geometry is the problem of optimally partitioning figures into simpler ones. Pioneers in this field mention at least two reasons for the interest :

(1) such a partition may give us an efficient description of the original figure, and
(2) many efficient algorithms may be applied only to simpler figures .

Besides inherent applications to computational geometry [C1], the partition problems have a variety of applications in such domains as database systems [LLMPL], VLSI and architecture design [LPRS] . Among others, the three following partition problems have been investigated :

MNRP (Minimum Number Rectangular Partition) . Given a rectilinear polygon with rectilinear polygon holes, partition the figure into a minimum number of rectangles.

MNCP1 (Minimum Number Convex Partition 1) . Given a polygon, partition it into a minimum number of convex parts .

MNDT1 (Minimum Number Diagonal Triangulation 1) . Given a polygon, partition it into a minimum number of triangles, by drawing not-intersecting diagonals .

In the above definitions, as in the course of the entire paper, we assume the following conventions . A *polygon* means a simple polygon (see [SH]), given by a sequence of pairs of integer-coordinate points in the plane, representing its edges. A *rectilinear polygon* is a polygon, all of whose edges are either horizontal or vertical . A *polygon with polygon holes* is a figure consisting of a polygon and a collection of not-overlapping, not-degenerate polygons lying inside it . The perimeter of the outer polygon and the contours of the inner polygons form *boundaries* of the figure, enclosing its *inside* equal to the inside of the outer polygon minus the boundaries and insides of the inner polygons . A *diagonal* of a planar figure is a line segment lying inside it and joining two of its non-adjacent vertices.

At first sight, MNRP and MNCP1 seem to be NP-hard. Surprisingly, both are solvable in time $O(n^3)$, where n is the number of corners of the input figure (see [LLMPL] and [C, CD]). The $O(n^3)$ time algorithm for MNRP uses a matching technique, that for MNCP1 is an example of a sophisticated dynamic programming approach. MNDT1 is also solvable in time $O(n^3)$, by a straightforward, dynamic programming procedure * *. In contrast to these results, we show the following problems to be NP-hard :

* This research was supported by NSF grants MCS-8006938 and MCS-7805849 .

** The known triangulation algorithm of time complexity O(nlogn) [GJPT] divides the input into n-2 triangles which is not always optimal [P].

PMNRP (Minimum Number Rectangular Partition for rectangles with *point* holes) . Given a rectangle with degenerate holes, i.e. isolated internal points, and a natural number k, decide whether the rectangle can be partitioned into k or fewer rectangles such that the points are not interior to any of the rectangles in the partition.

MNCP (Minimum Number Convex Partition) . Given a polygon with polygon holes, and a natural number k, decide whether the figure can be partitioned into k or fewer convex parts.

3MNCP1 (Three Dimensional Minimum Number Convex Partition 1) . Given a one-connected polyhedron and natural number k, decide whether the polyhedron can be partitioned into k or fewer convex parts.

Chazelle later focus on this

MNDT (Minimum Number Diagonal Triangulation) . Given a polygon with polygon holes, and a natural number k, decide whether the polygon can be partitioned into k or fewer triangles, by drawing not-intersecting diagonals.

MNT (Minimum Number Triangulation) . Given a polygon with polygon holes, and a natural number k, decide whether the figure can be partitioned into k or fewer triangles.

The NP-hardness of 3MNCP1 explains why Chazelle was able to develop only approximation polynomial-time algorithms for this problem [Cl].

The PMNRP problem allows point holes , i.e. degenerate polygon holes. The idea of point holes is not quite abstract . For instance, if we divide some area full of holes into rooms without holes, drawing lines of standard thickness δ, then holes of dimensions not exceeding δ may be viewed as point holes.

PMNRP and MNDT can easily be shown to be in NP. The membership of the three remaining NP-hard problems in NP is an open question .

The NP-completeness of PMNRP suggests that point holes are harder than rectilinear polygon holes . Similarly, the second and the fourth NP-hard result suggest that multiconnected polygons are much more difficult to decompose than one connected ones. In the proof of NP-hardness of MNCP, MDNT,and MNT strongly non-rectilinear holes play an important role. This, and the fact that point holes may also be viewed as non-rectilinear holes, explains the title .

It is interesting that if we look for a minimum edge length rectangular partition then rectilinear polygon holes are sufficient to obtain NP-completeness. The minimum edge length problems corresponding to the NP-hard minimum number partition problems are the more NP-hard (see [LPRS]).

This paper is an improved version of an original draft with the same title. The first reason for this improvement has been a recent paper of O'Rourke and Supowit [OS]. They obtained three NP-hardness results for minimum number decomposition problems, allowing overlapping of decomposing figures. Their proofs are by transformation from 3SAT, whereas we use a planar version of 3SAT which has been recently shown to be NP-complete by Lichtenstein [L] . If O'Rourke and Supowit knew about Lichtenstein's result, they could eliminate overlapping, which they used only in the design of crossovers. Taking this into consideration, their results coincide with ours in the case of the NP-hardness of MNCP. The optimal partitions of the multiconnected polygon, constructed by O'Rourke and Supowit in their proof of the NP-hardness of the minimum number convex decomposition problem, can be obtained by drawing not-intersecting diagonals. *Hence their proof technique (contrary to ours) also yields the NP-completeness of the minimum number diagonal convex partition problem.* In our original draft, truth setting components are unneccessarily complicated. Here they are reduced to simple variable loops, following the idea of O'Rourke and Supowit. The second reason has been the achievement of new results, i.e. the NP-hardness of MNDT, and MNT. In their proof, we again use ideas from [OS].

NP-hardness of PMNRP and MNCP

We shall assume a slightly less restricted version of planar 3SAT, PL3SAT, with the following instances :

3CNF formula F with variables x_i, $1 \le i \le n$, and clauses c_j, $1 \le j \le m$, and a planar bipartite graph $G=(\{x_i \mid 1 \le i \le n\} \cup \{c_j \mid 1 \le j \le m\}, E)$ such that $(x_i, c_j) \in E$ if and only if x_i or $\bar{x}_i$ is a literal of c_j.

To prove the NP-completeness of PMNRP we shall reduce a slight modification of PL3SAT to a generalization of PMNRP.

In comparison to PL3SAT, the modified PL3SAT (MPLSAT) allows arbitrary clauses consisting of two literals, but on the other hand, each clause with three literals has to contain at least one negated, and one positive literal. By adding new variables we can easily reduce PL to MPLSAT. Thus MPLSAT is NP-complete.

By a *rectilinear figure* we shall mean a polygon with holes in the form of *rectilinear polygons with rectlinear polygon holes, vertical or horizontal line segments, and isolated points,* where the inside polygons and line segments do not intersect but may touch one another. Clearly, the inside of a hole in a hole of a rectilinear figure is a part of the inside of the figure.We consider the following generalization of PMNRP :

GMNRP . Given a rectilinear figure, and a natural number k, decide whether the figure can be partitioned into k or fewer rectangles.

By *concave points* of a planar figure we shall mean not only the corners of its interior, reflex angles, but also its point holes, and the endpoints of its line segment holes. Depending on the context, we shall understand a *partition* of a figure into simpler ones either as the collection of the partitioning line segments or as the set of the simpler figures. The following lemma is an obvious generalization of Theorem 1 from [LLMPL].

Lemma 1 . In any minimum number partition of a rectilinear figure into rectangles, each line segment is colinear with a concave vertex of the figure.

We can simulate the boundaries of internal rectilinear polygons and segments by appropriate dense points. This, Lemma 1 , and the fact that we can find an optimal partition for each of the polygon holes of a rectilinear figure in polynomial time (see Introduction) yields :

Theorem 1 . GMNRD is many-one polynomial-time reducible to PMNRP.
Proof. Let F be a rectilinear figure. We may assume w.l.o.g. that the outer boundary of F is a rectangle. Otherwise we can easily construct a rectangle with a rectangular hole such that F can be embedded in the hole. The area between the rectangular boundary of the hole and the outer boundary of F forms a multiconnected hole in the resulting figure. Clearly, F can be partitioned into k rectangles if and only if the resulting figure can be partitioned into k+4 rectangles.

The lines colinear with boundary segments of F induce a rectilinear grid with at most n^2 grid points inside

F or on boundaries of F. Hence, we can partition F into n^2 or fewer rectangles. Between each pair of neighboring horizontal or vertical lines of the grid, let us respectively draw n^2 horizontal or vertical new lines . In other words, we embed the original grid in a new grid, n^2+1 times thinner. Let F' be the figure of the same external boundary as F, containing as point holes all the points of the new grid that lie on internal boundaries of F.

We shall show that there exists an optimal partition of F' into rectangles, containing all line segments lying on internal boundaries of F. This will prove that F can be partitioned into k or fewer rectangles if and only if F' can be partitioned into k+m or fewer rectangles, where m is the number of rectangles in a minimum number rectangular partition of the *polygons with polygon holes* that are holes in F. By applying the mentioned algorithm for MNRP, we can determine m in polynomial time. Hence, we shall obtain polynomial-time reducibility of GMNP to PMNRP .

Assume inductively that there is a minimum number rectangular partition of F', M, including all internal boundary segments of F lying on the first i-th bottom, horizontal grid lines plus k internal boundary segments lying on the i+1 line from the bottom . Notice that by Lemma 1 M lies on the new grid . It is sufficient to show how to construct a new optimal partition of F, additionally including one more of such horizontal segments on the i+1 line . When we have an optimal partition of F', including all horizontal boundary segments, we can repeat this inductive procedure for vertical segments.

Let s be such a horizontal segment not included in M , and let C be the collection of all new grid points that lie inside s, but not inside any horizontal segment of M. If C is empty then we are done . The number of rectangles in M is not greater than the number of new grid points inside s. For this reason, there are p, q ∈ C, such that p, q are neighbors in the new grid and p is an endpoint of a horizontal segment in M, disjoint from q (see Fig.1).

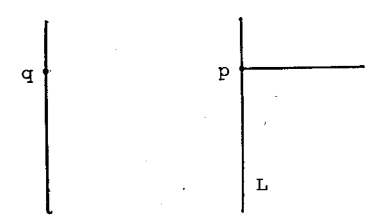

Fig.1.

Let L be the vertical line segment containing p. If L is colinear with a concave point of F, then we mark the pair (p,q) and look for another such a pair, unmarked as yet. Otherwise, we find the closest to p, horizontal segment of M or of the perimeter of F, that lies below p and touches L from both sides. Let u be the piece of L lying between p and the above segment . We move u towards the vertical line of q, pulling all horizontal segments touching u from the other side, and compressing the horizontal segments between u and the vertical line of q. Since p and q are neighbors in the new grid, we can not meet any vertical segment during this movement (see Lemma 1), before reaching the line of q. As a result, we obtain a new optimal partition of F', still including the same horizontal segments, with the set C decreased by p. Iterating this process we come to the situation where only marked pairs (p,q) may exist . If C is empty then we are done . Otherwise, there are two points of the original grid, lying on s, such that at least n^2 vertical segments lie between them in the partition of F', recently constructed. This contradicts the minimality of this partition ∎

In the proof of Theorem 1 , the density of the simulating points is essential . For instance, if we cover a tall tower of rectangular holes with too scattered colinear points then it might be more efficient to draw only a series of vertical lines passing through these points instead of drawing the boundaries of the rectangles .

Now it is clear that a reduction of MPLSAT to GMNRP implies the NP-completeness of MNRP.

Let (F, G) be an instance of MPLSAT, where F is a formula and G is the corresponding planar graph . To reduce MPLSAT to GMNRD we construct a rectilinear figure , and a natural number k such that F is satisfiable if and only if the figure, denoted by H, can be decomposed into k or fewer rectangles.

The basic component of H is a cranked wire (see Fig.2 (A)) .The dimensions of the cranks are not essential . Only the colinearity of segments is important. Each wire is several times bent 90° to form a closed loop. A straight section of wire needs to contain one or two cranks (See Fig. 2 (A)) . By applying isolated points we could even have a simpler form of wires . However this would decresase the uniformity of our proofs. We have the following, obvious lemma :

Lemma 2 . A separated wire loop is most efficiently partitioned into rectangles either horizontally or vertically but not both (see Fig. 2 (B,C)). Any other partition yields at least one rectangle more.

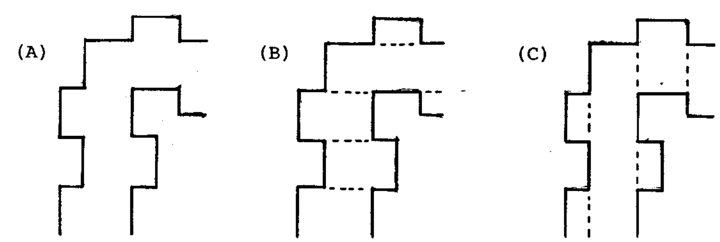

Fig.2. A section of a wire with two bends (A), and the vertical and horizontal partition of the section (B,C) .

Each variable x corresponds in a one-to-one manner to a wire loop. We interprete the (absolute) vertical decomposition of the loop as setting x to 1, and the horizontal decomposition as setting x to 0 .

Each clause c corresponds in a one-to-one manner to a junction. Three and two argument clause junctions may occur in H . A three argument junction is shown in Fig. 3 (A) . Two argument junctions can be obtained by blocking one of the arms of the triple junction . The c-junction touches a loop bend (see Fig.3 (A)) if and only if the variable x corresponding to the loop appears in c . Loop bends touched by c junction correspond in a one-to-one manner to literals of c . The arm of such a bend that lengthens the junction , is vertical if x is a literal in c, and is horizontal if $\bar{x}$ occurs in c. If the c-junction is three argument, the above requirement can always be realized due to the fact that c contains at least one positive , and one negative literal. Owing the planarity of G, junctions and loops are arranged in such a way that they do not overlap.

Note that if the arm of a loop bend which lengthens a junction is partitioned by segments parallel to its direction then the long rectangle inside it can be expanded inside the junction. For instance, see

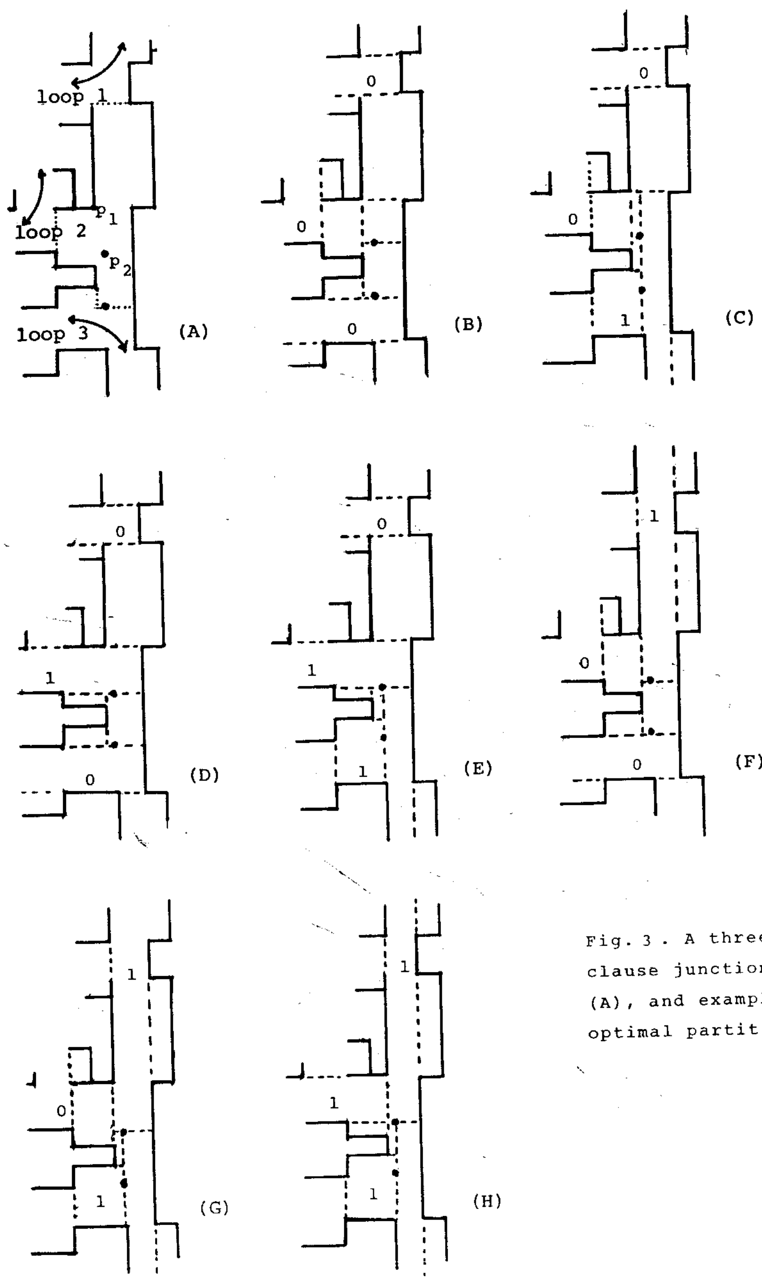

Fig. 3 . A three argument clause junction for GMNRP (A), and examples of its optimal partitions (B-H).

Fig. 3 (C, D) . By the orientation of the junction and Lemma 1, such a partition of the wire section means that the literal corresponding to the loop bend has the value 1 under the variable setting given by loops . This all enables us to prove :

Lemma 3 . If at least one loop adjacent to a clause junction brings 1 for the corresponding literal, then we can lengthen some rectangles occurring in the adjacent loops such that only 3 new rectangles are needed to partition the junction. Otherwise, at least 4 new rectangles are necessary .

The points p_1 , p_2 in Fig. 3 (A) are not horizontally colinear. Two rectangles coming from the bottom and top wire respectively are thus prevented from merging. Therefore, we can also prove :

Lemma 4 . In any partition of H, at least 3 rectangles lie wholly inside a clause junction.

Let k be the total minimum number of rectangles partitioning loops plus 3 times the number of clauses of F. It follows from Lemmas 2 through 4 that any partition of H can have no more than k rectangles only if F is satisfiable . On the other hand, if F is satisfiable then by Lemmas 2 and 3, we can cover H with k not overlapping rectangles. It suffices to partition the loops according to a 0, 1 assignment that satisfies F and to partition each clause junction using only three inside rectangles .

Lemma 5 . H can be partitioned into k or fewer rectangles if and only if F is satisfiable.

The construction of H can be performed in logarithmic-space. By Lemma 1 we may consider only these rectangular partitions of H , in which each edge is colinear with a concave vertex of F . Finally, the dimensions of H and the number k are polynomially related to the size of G. Summarizing :

Theorem 2 . GMNRP is strongly NP-complete .

By Theorem 1 we obtain :

Corollary 1 . PMNRP is strongly NP-complete .

In the case of MNCP, point holes are not allowed . Therefore, to prove the NP-hardness of MNCP we have to modify H . A new three argument junction is shown in Fig. 4 (A) . The sharp "sprouts" replace the isolated points and penisular segments of the old junction. Fig.4 (B) through (H) shows optimal partitions of the junction . The absence of point holes does not mean that the constructed figure does not have holes at all. First of all, any loop creates an island . We could even get rid of the islands surrounded by loops, using unclosed wires with odd number of bends, instead of the wire loops. However, if the planar G contains cycles then polygon one-connected islands surrounded by wires and junctions will still appear in the figure. Analogously we can prove:

Theorem 3 . MNCP is strongly NP-hard .

The details of the proof of Theorem 3 are left to the reader .

Corollary 2 . 3MNCPI is strongly NP-hard .

Proof. The proof is by a reduction of MNCP to 3MNCPI. We transform the input polygon with polygon holes into a polyhedron consisting of two horizontal layers. The cross-sections of the first layer are equal to the input, multiconnected polygon. The cross-sections of the second layer are equal to a fixed rectangle whose horizontal

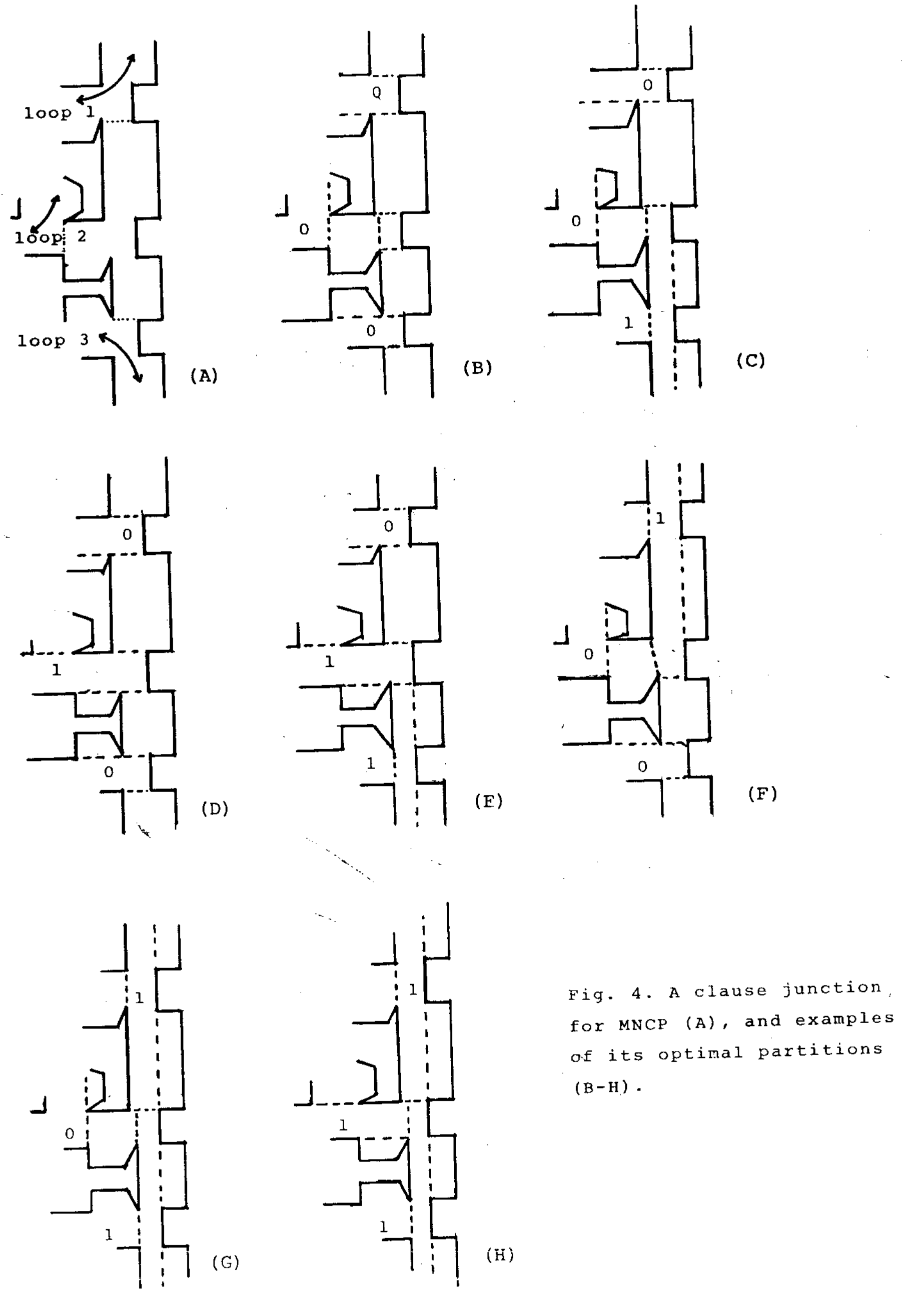

Fig. 4. A clause junction for MNCP (A), and examples of its optimal partitions (B-H).

projection includes the horizontal projection of the first layer. We may assume w.l.o.g. that the input polygon contains at least one hole. Therefore, it can be partitioned into k convex polygons if and only if the polyhedron can be partitioned into k+1 convex parts ∎

As in the rectilinear case, we could simulate the boundaries of polygon islands by dense points, obtaining as a corollary the NP-hardness of the problem of partitioning polygons with point holes into convex polygons. However the proof of this fact is much longer than that in the rectilinear case, and therefore we shall skip it.

NP-hardness of MNDT and MNT

The proof of NP-hardness for MNDT and MNT is by a direct reduction from PL3SAT. Let F be a 3CNF formula, where the bipartite graph G associated with F is planar. We shall construct the polygon with polygon holes, H, and the natural number k such that :

(i) H can be partitioned into k or fewer triangles if and only if F is satisfiable, and
(ii) there is a minimum number partition of H into triangles, where all edges of the triangles are diagonals of H.

Since the construction of H can be performed in log-space, the two above properties of H will imply NP-hardness of MNDT and MNT.

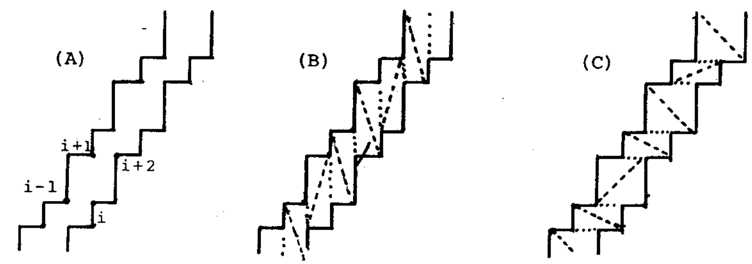

Fig.5. A straight section of a loop for MNDT and MNT (A), and its optimal partitions into triangles (B,C).

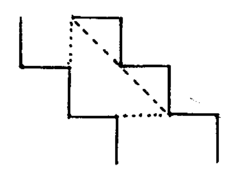

Fig.6. A triangulation of a piece of Masek's wire.

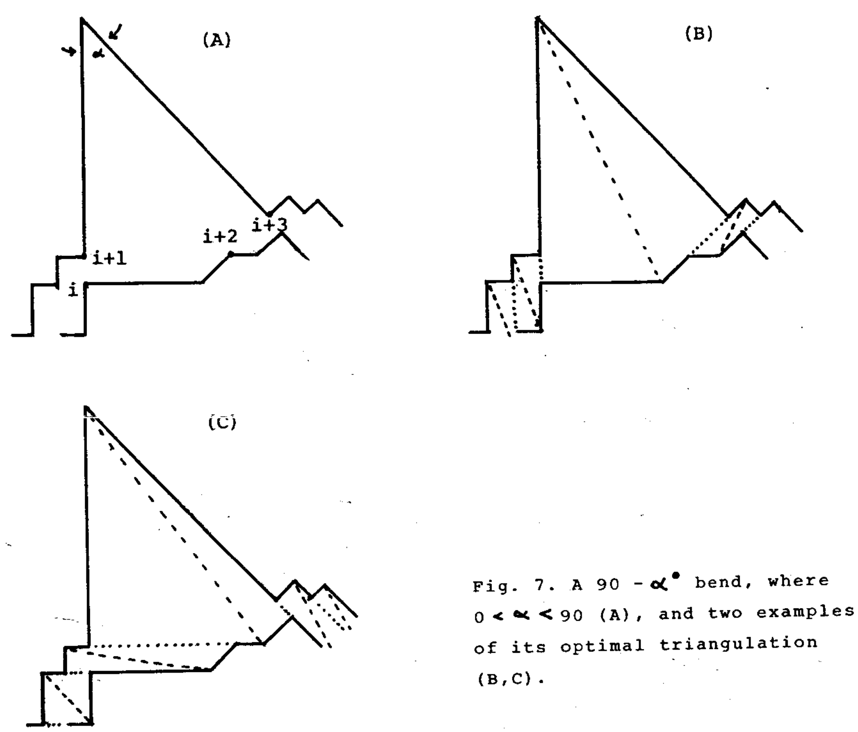

Fig. 7. A 90 - α° bend, where $0 < \alpha < 90$ (A), and two examples of its optimal triangulation (B,C).

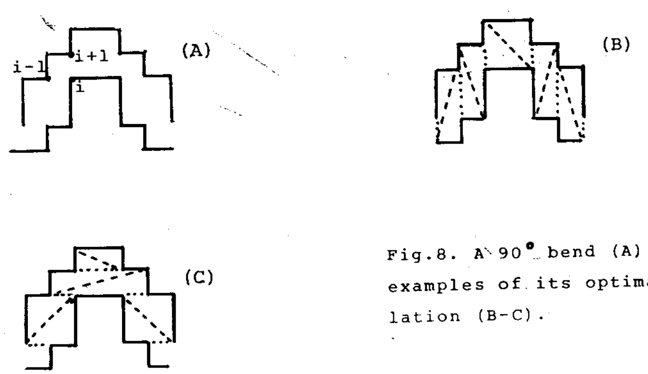

Fig.8. A 90° bend (A), and two examples of its optimal triangulation (B-C).

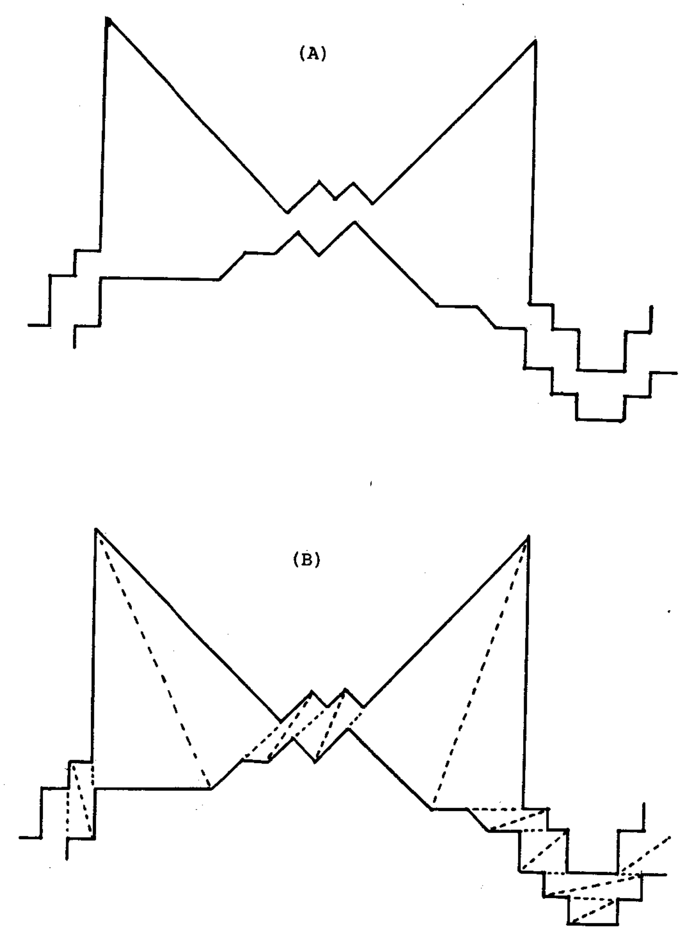

Fig.9. A signal invertor (A), and an example of an optimal triangulation of the invertor (B).

The design of H is similar to that from the previous NP-hardness proof . H again consists of variable loops and clause junctions . A straight section of a variable loop (see Fig. 5(A)) is similar to the wire orginally applied by Masek [M] , and then by O'Rourke and Supowit (see Fig. 6). Our modification consists in alternating the length of the vertical segments bounding such a section (assuming that it forms 45° angle with the horizontal direction). Due to this change, we can eliminate the possibility of the triangulation shown in Fig. 6. There remain two optimal methods of partitioning such a section into triangles (see Fig. 5(B,C)). We can connect pairs of its interior concave vertices either by horizontal or by vertical line segments. In this way we divide the section into rectangles. Then, it is not essential which diagonals are used to divide the rectangles into triangles . The horizontal method is interpreted as transmitting 1 and the vertical method as transmitting 0.

Loops may bend any angle between 0° and 90° (see Fig. 7) . This kind of bend is a modification of a bend invented by O'Rourke and Supowit [OS] . See Fig. 7 (B) and (C) for two ways of optimally partitioning such a bend into triangles . We shall also use another, 90° bend shown in Fig. 8 (A). Two 45° bends combined with this 90° bend invert the signal (see Fig. 9). The 90° bends are optional in the construction of invertors. They can be eliminated by deforming the square sides of loops.

Let us number the interior concave corners of a separated variable loop according to Fig. 5 through 8 , using consecutive natural numbers 1 through 2n. We obtain the following lemma :

Lemma 6 . In any minimum number partition of the loop into triangles, either each pair of concave vertices 2k, 2k+1 mod 2n, or each pair of concave vertices 2k-1, 2k, is connected by a diagonal.
Proof: Simultaneously, let us draw diagonals between each pair of concave vertices 2k-1, 2k , and each pair of concave vertices 2k, 2k+1 mod 2n. Next, let us erase these diagonals that lie inside 90° bends. As a result, we obtain a partition of the loop into a collection of quadrangles and 90° bends, say D (see Fig. 10). Consider any partition of the loop into triangles, say T. For each triangle t∈T, let n(t) be the number of elements from D whose inside overlaps with the inside of t. Given d∈D, we define T(d) as the sum of $n(t)^{-1}$ over all triangles t whose inside overlaps with the inside of d. It is clear that T partitions the loop into exactly $\Sigma_{d \in D} T(d)$ triangles.
Lemma 6 results from the two following observations :

(i) for each quadrangle from D, the minimum value of T(d) is 1, and for each 90° bend the minimum value of T(d) is 4, and
(ii) the only way to achieve the above minimum values is to connect by diagonals either each pair of vertices 2k, 2k+1 mod 2n, or each pair of vertices 2k-1, 2k ∎

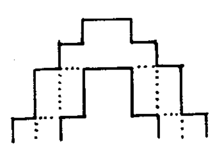

Fig.10. A partition of a loop section into quadrangles and 90° bends.

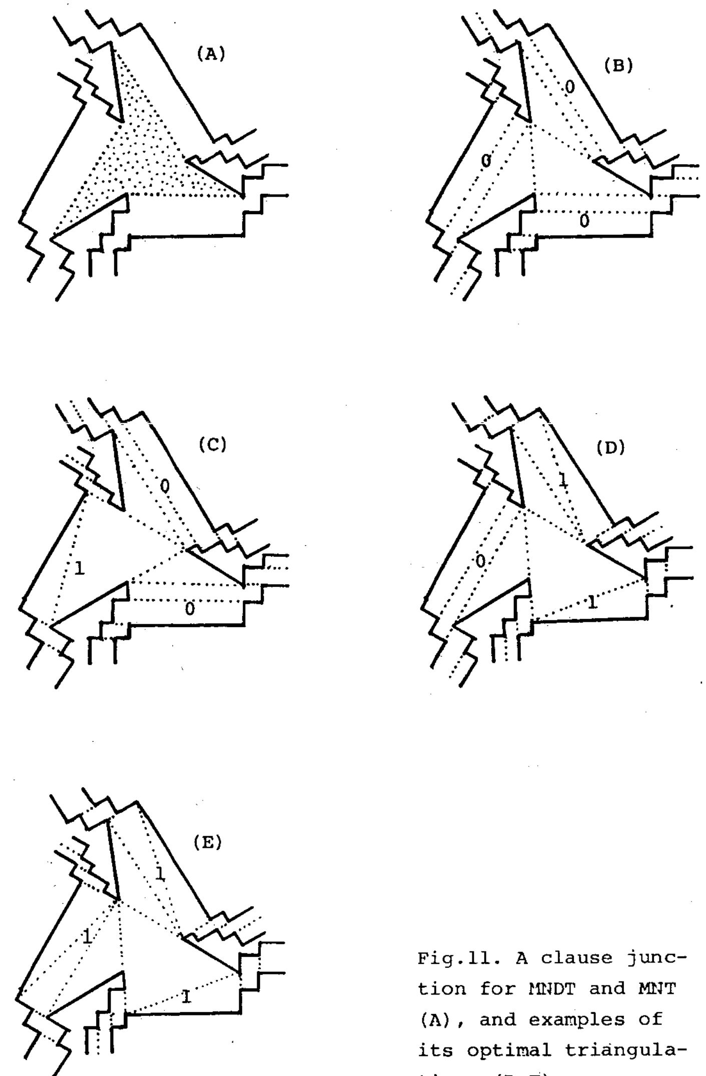

Fig.11. A clause junction for MNDT and MNT (A), and examples of its optimal triangulations (B-E).

Each straight section of a variable loop that is adjacent to a clause junction, corresponds to a negative or positive occurrence of the variable represented by the loop , in the clause represented by the junction . Signal invertors are installed in each variable loop in such a way that by Lemma 6 each loop has the following property :

Lemma 7 . In any minimum number partition of a separated variable loop into triangles, each of its straight sections adjacent to a clause junction is partitioned in either the relative horizontal or the relative vertical way . Any of two such sections are partitioned in the same horizontal or vertical way if and only if both of them correspond either to a negative or to a positive occurrence of the variable represented by the loop .

A clause junction is shown in Fig. 11 (A) . By examining Fig. 11 (B) through (E) we obtain the following lemma on clause junctions :

Lemma 8 . If at least one of the three sections of variable loops adjacent to a clause junction is partitioned horizontally then only two additional triangles are needed to partition the area of the junction . Otherwise, exactly three additional triangles are needed . In both cases, the line segments partitioning the junction may be restricted only to the diagonals of H.

Let us define k as the minimum number of not overlapping triangles necessary to cover separate variable loops plus twice the number of clauses of F . By Lemmas 7 and 8 we obtain :

Lemma 9 . F is satisfiable if and only if H can be partitioned into k or fewer triangles.

By Lemmas 6 and 8 , and the design of H, there is always a minimum number partition of H which is a diagonal triangulation of H, i.e. all line segments in such a partition are diagonals of H. This yields the following, modified version of Lemma 11.

Lemma 10 . F is satisfiable if and only if there is a triangulation of H consisting of k or fewer triangles .

Similarly, as Lemma 5 implies Theorem 2, Lemma 9 and 10 imply :

Theorem 4 . MNDT and MNT are strongly NP-hard .

Corollary 2 . MNDT is strongly NP-complete .

Final Remarks

From the application point of view, it is not so important to have a partition achieving the minimum number of parts. A nearly optimal partition is quite sufficient. The author believes that there are good approximation heuristics for the minimum number problems corresponding to the decision problems shown to be NP-hard in this paper.

In the proof of the NP-completeness of PMNRP is essential to allow point holes to be *corectilinear*, i.e. two or more point holes may occur on the same horizontal or vertical line. Open is the restricted version of PMNRP where the only holes allowed are non-corectilinear points [P].

Acknowledgements

I would like to express my appreciation to Ron Pinter, Ronald Rivest and Adi Shamir for their encouragment, to Tom Leighton for telling me about the NP-completeness of planar satisfiability, and to David Johnson for his critical remarks.

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