Jesus.

I now know the answer to one of the questions that I asked you in my previous note.

Given an $n\times n$ P matrix M, it is not always the case that the 2^n complementary ~~matrices~~ cones formed from the columns of $[I, M]$ have nonempty n-dimensional intersection. So it could be (for $n\geq 3$) that there is no chamber contained in the intersection of the 2^n complementary cones.

However, there is always a virtual chamber. The reason is that if M is a P-matrix, then M^T is also a P-matrix. The chambers from $[I, M]$ correspond to triangulations of $[-M^T, I]$. It is a theorem from Samelson, Thrall & Wesler that the complementary cones of $[-M^T, I]$ partition n-space iff M^T is a P-matrix.

Thus, if M (and hence M^T) is a P-matrix, then there is a triangulation (partition into simplicial cones) of $\mathbb{R}^n$ in which the maximal faces are precisely the complementary cones of $[-M^T, I]$. (This triangulation looks like a cross-polytope)

This means that we have the following situation, given $M \in \mathbb{R}^{n\times n}$: (better $\mathbb{Q}^{n\times n}$)

The question: "Is there a triangulation of $\mathbb{R}^n$ that uses all of the complementary ~~simplices~~ (cones) of $[-M^T, I]$?" is NP-hard because it asks if M is a P-matrix.

The question: "Is there a regular triangulation of $\mathbb{R}^n$ that uses all of the complementary cones of $[-M^T, I]$?" has a polynomial-time ~~solution~~ algorithm by my paper "Geometric Properties of Hidden Minkowski matrices".

Interesting, huh? In both of these cases the input is M, and not something of size $\binom{2n}{n}$ (such as an ~~e~~xplicit description of the objective function for the optimization over the universal polytope).

I am starting to think that it's not clear if the right input should be just the point set or an explicit description of the set of simplices we want to count ~~Defi~~ ~~Sometimes~~ The ~~~~ answer seems to differ from problem to problem.

Best Wishes, Walter

P.S. I saw Christos Athanasiadis at George Washington U. on Friday. He gave (among others!) a talk about the generalized Baues problem. I asked him if he thought that the number of triangulations of a ~~d dimensional 2 point~~ set of $2d$ points in $\mathbb{R}^d$ could be doubly exponential, and he said he didn't think so, but it seemed as though he hadn't thought ~~of it~~ about it.

(I should have said $\mathbb{R}^{d-1}$ instead of $\mathbb{R}^d$)

The point is that Billera–Filliman–Sturmfels show that the number of <u>regular</u> triangulations is $O((2d)^{d^2})$, which is not doubly exponential, but on the other hand there is this general feeling that the number of all triangulations is much bigger than the number of regular triangulations.

If you see him, (Athanasiadis) ~~ple~~ tell him I enjoyed listening to his $\underline{3}$ talks. I didn't get a chance to tell him because I had to hurry home after the talks.