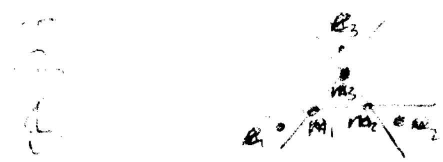
Dear Jesus,

Here are some reprints of papers of mine. The ones that I think that you would find most interesting are the papers "Geometric Properties of Hidden Minkowski Matrices," and "Extended P-pairs".

Let M be an $n \times n$ matrix. The "Hidden Minkowski" paper studies a property $(*)$, which means that there exists a vector p (a "PC-point") that is in the interior of the intersection of all of the complementary cones generated by $[I, M]$. (Remember that the usual LCP setup looks at the cones coming from $[I, -M]$, so this is slightly different). The main result of that paper is that this chamber if it has nonempty interior, is ~~a~~ simplicial cone (has the face lattice of a simplex). Also, one can check if this is true in polynomial time.

<u>E.g.</u> $M = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{bmatrix}$ then the points of $[I, M]$ look like



and the chamber we're interested in is the cone generated by the columns of M.

On the other hand, if $M = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}$, then there is no such chamber – the intersection of the complementary cones generated by $[I, M]$ is the origin (even though $\begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}$ is a P matrix)

So, there exist P matrices M for which there is no PC-point.

This means that the intersection of the complementary cones is not a chamber. Could it be a <u>virtual</u> chamber? I think that there must be examples of P matrices for which the intersection of the complementary cones is not even a virtual chamber, but I haven't gotten around to looking for them.

The ~~[illegible]~~ algorithm for determining if there is a chamber that meets all of the complementary simplices is polynomial even in the size of M, so it is an example that shows that maximizing a linear functional over the secondary polytope ~~[illegible]~~ is not always hard.

But is the situation for the universal polytope any different? I.e. ~~[illegible]~~ Are there examples in which the intersection of the complementary simplices is a virtual chamber, but not a real chamber, and is it hard to determine if this is the case?

Thanks again for a very enjoyable visit, and best regards to your family.

Walter