Prop: $A \subseteq \mathbb{R}^d$ a set of points s.t. any subdivision of A has a regular coarsening. Then Baues for A-triangulations is true.

proof: Let P = Baues poset (= proper subdivisions ordered by refinement)

$\uparrow$

Q = regular Baues subposet (= face poset of $\partial\Sigma(A)$) secondary polytope

Define $f: P \to Q$ as follows:

given a subdivision p, you get a set of strict inequalies on the height variables $(h_1, h_2, \dots, h_{|A|})$ that would induce p. Change all the strict ineqs to weak ineqs, and the solution set will be a closed cone σ in the secondary fan.

Claim 1: Since p has a regular coarsening p', the closed cone σ will not be $\underline{0}$, because $\sigma \geq \sigma'$ where σ' is the cone for p'

Now since σ is a closed cone in the secondary fan,

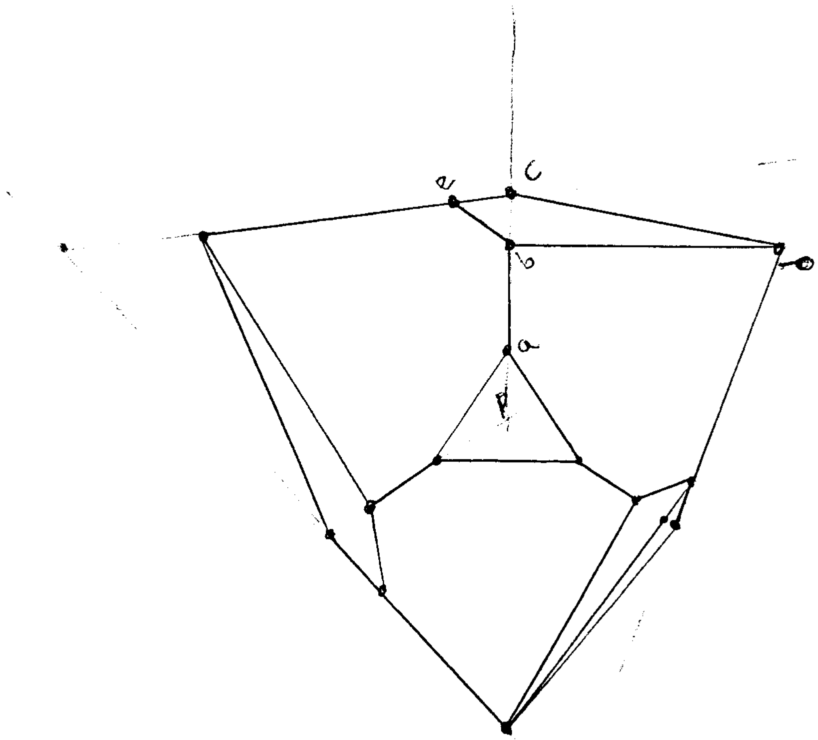
The trick point; is this true? → it lies in the relint of a cone corr. ~~corresponds~~ to some reg. subdivision q.

Set $f(p) = q$.

Claim 2: $f: P \to Q$ is order-preserving.

Check Quillen condition: Given $q \in Q$, is $f^{-1}(Q_{\leq q})$ contractible in P?

Claim 3: $f^{-1}(Q_{\leq q}) = P_{\leq q}$ ∎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