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This certification is a material representation of fact upon which reliance was placed when this transaction was made or entered into. Submission of this certification is a prerequisite for making or entering into this transaction imposed by section 1352, title 31, U.S. Code. Any person who fails to file the required certification shall be subject to a civil penalty of not less than $10,000 and not more than $100,000 for each such failure.

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A. Project Summary

Descartes' Rule of Signs bounds the number of real roots of a polynomial $f(x)$ in terms of sign pattern of the coefficients of its monomials. Recently, I. Itenberg and M. -F. Roy conjectured a multidimensional generalization of Descartes' rule of signs by means of combinatorial constructions of a polynomial system. This conjecture attracts considerable attention in the community and is known to be true in some special cases. Very recently, we announced an example which disproves the conjecture. Based on the insight the counter example provides, we propose to make a revision of the combinatorial upper bound they formulated. The main goal of this project is to fulfill all the theoretical gaps of our new combinatorial upper bound. It is hoped that the proposed research will result in a proper generalization of Descartes' rule of signs in higher dimensions.

Note: The project description was written jointly with Professor T. Y. Li of Michigan State University who is submitting the same project description in a simultaneous project from that institution.

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C Project Description

The famous Descartes' Rule of Signs says that the number of positive real zeros of a polynomial $p(x)$ is bounded above by the number of its sign changes in the list of its coefficients. As an example, consider

$$p(x) = x^{203} - x^{100} + 1.$$

It has 203 distinct zeros. By Descartes' Rule of Signs, there are at most two positive real roots and (by replacing x by $-x$) at most one negative zero. Similar to the result of this univariate Descartes' rule, the following question arises:

(*) *What is the maximum number of isolated real solutions of any system of n real polynomial equations in n variables which does not depend on the degrees of the given equations?*

Writing R_+ for the set of positive real numbers, we have the following general conjecture toward this question by Kuchnirenko:

Any system of n real polynomial equations in n unknowns where the i-th equation has at most m_i terms can have at most $(m_1 - 1) \cdots (m_n - 1)$ isolated roots in $(R_+)^n$.

Obviously, this conjecture is true when $n = 1$. The number of real positive zeros of a univariate polynomial is bounded above by the number of terms minus one—an immediate consequence of Descartes' Rule of Signs. In fact, this is the only case where Kuchnirenko's conjecture is known to be true, it remains open in all higher dimensions.

A major breakthrough on the problem (*) was accomplished by Khovanskii (1980):

Theorem 1.1 *(Khovanskii) For a system of n real polynomial equations in n variables involving k distinct monomials in total, the number of isolated roots in the positive orthant $(R_+)^n$ is $2^{\frac{k(k+1)}{2}}(n+1)^k$.*

This theorem gives an upper bound for the root count in $(R_+)^n$. If we wish to consider roots in $(R^*)^n$, where $R^* = R \setminus \{0\}$, instead of $(R_+)^n$, then we simply multiply the asserted bound by 2^n and the theorem implies an upper bound for the root count suggested in Problem (*). But this bound exceeds the Bézout number in most occasions and can be frustratingly large even for very small systems. For instance [7], for a system of two polynomial equations in two unknowns with four terms each, this bound will reach 4,586,471,424 in $(R_+)^2$, or 18,345,858,696 in $(R^*)^2$. The bound according to Kushnirenko's conjecture is only 36. This huge difference makes Kushnirenko's conjecture seem somewhat optimistic.

One approach to refine Kushnirenko's conjecture is to take Newton polytopes of the given polynomial systems into account. Let $p_1, ..., p_n$ be real polynomials in n variables, and $A_1, ..., A_n$ and $\Delta_1, ..., \Delta_n$ be the supports and Newton polytopes of these polynomials respectively. Each support A_i can be equipped with a distribution δ_i of signs at its integer points: a point gets the sign ("+" or "−") of the coefficient of the corresponding monomial of the polynomial p_i. Each polytope Δ_i with a distribution δ_i on A_i is called a *signed Newton diagram.*

Let ω_i be a real-valued function defined on the set A_i. By taking the lower convex hull in R^{n+1} of the graph of ω_i and then projecting each facet to $R^n \times \{0\}$, the function ω_i, usually called *a lifting* on A_i, induces a polyhedral subdivision τ_i of Δ_i. Denote by Δ_M the Minkowski sum of the polytopes $\Delta_1, ..., \Delta_n$ and by A the set

$$\{a \in \Delta_M | a = a_1 + ... + a_n, \; where \; a_i \in A_i\}.$$

Define a function $\omega : A \longrightarrow \mathbf{R}$ as:

$$\omega(a) = min\{w_1(a_1) + ... + \omega_n(a_n)\}$$

for $a_1 + ... + a_n = a$. Such a function ω defines a polyhedral subdivision τ_ω of Δ_M. The vertices of τ_ω belong to A and each polytope F of τ_ω has a unique representation

$$F = F_1 + ... + F_n,$$

where F_i is a facet of τ_i. Suppose that the functions $\omega_1, ..., \omega_n$ are generic, then for any polytope $F = F_1 + ... + F_n$ of τ_ω, we have

$$dim(F) = dim(F_1) + ... + dim(F_n).$$

A subdivision τ_ω of Δ_M obtained this way is called *mixed subdivision.* A polytope $V = v_1 + ... + v_n$ of τ_ω such that

$$dim(v_1) = ... = dim(v_n) = 1$$

is called a *mixed cell.* We call a mixed cell $V = v_1 + ... + v_n$ *alternating* if the distribution of signs δ_i associates different signs to the end points of v_i. For any mixed subdivision τ_ω, we can count the number of alternating mixed cells. We define the *combinatorial upper bound* for the system $p_1, ..., p_n$ to be the maximum number of alternating mixed cells in any mixed subdivision τ_ω of Δ_M.

Based on Sturmfels' work [8] on Viro's method [9], Ilia Itenberg and Marie-Francoise Roy [3] conjectured that the number of real positive isolated zeros of any real polynomial system $p_1, ..., p_n$ in n unknowns is bounded above by the combinatorial upper bound. This conjecture attracted considerable attention in the community and is known to be correct in a few special cases. In particular, for one polynomial in one unknown it coincides with Descartes' Rule.

To raise even more awareness of the Itenberg-Roy conjecture, B. Sturmfels proposed at Mathematisches Forschcongsinstitat Oberwolfach a special case as a challenge problem, and offered a $500 reward for its solution [5]. The problem is to resolve the conjecture for the system

$$\begin{aligned} x^5 &= a_1 y^5 + a_2 x^3 y^5 + a_3 x^6 y^8 \\ y^5 &= b_1 x^5 + b_2 x^5 y^3 + b_3 x^8 y^6 \end{aligned}$$

with $a_1, a_2, a_3, b_1, b_2, b_3 > 0$. And the award was paid to J. C. Lagarias and T. J. Richardson for solving it [5].

We felt that the Itenberg-Roy conjecture tended to be somewhat local. From the point of view of the polyhedral homotopy $H(x, t) = 0$ introduced by B. Huber and B. Sturmfels [2], the root count is correct for systems $H(x, t) = 0$ when t is very close to zero. However, many other phenomena may occur globally. Based on this, we constructed an example [6] very recently which disproved the Itenberg-Roy conjecture. Our example consists of two equations in two unknowns:

$$\begin{aligned} p_1(x_1,x_2) &= x_2 - x_1 - 1 \\ p_2(x_1,x_2) &= x_1^3x_2^3 + 100x_1^3 - 900x_2^3 - 200. \end{aligned} \tag{1}$$

Here the combinatorial upper bound is only two [6], but there are three positive real roots:

$$a = (0.3177, 1.3177), b = (0.6600, 1.6600), c = (8.1206, 9.1206).$$

Nevertheless, we believe that Itenberg and Roy's work is very valuable. It is on the right track and a further investigation on a *modified* Itenberg-Roy conjecture we formulate below may provide an appropriate Multivariate Descartes' Rule of Signs:

Let's look at the conjecture more closely. Let $V = v_1 + \cdots + v_n$ be a mixed cell of the subdivision τ_ω, and for $1 \le i \le n$ let a_i and b_i be the endpoints of v_i. Then with $\mathrm{sign}(a_i)$ and $\mathrm{sign}(b_i)$ as assigned by the distribution δ_i, the system of equations,

$$\begin{aligned} \alpha_1 x^{a_1} + \beta_1 x^{b_1} &= 0 \\ &\vdots \\ \alpha_n x^{a_n} + \beta_n x^{b_n} &= 0 \end{aligned}$$

where $sign(\alpha_i) = sign(a_i)$ and $sign(\beta_i) = sign(b_i)$ for $1 \le i \le n$, is called a *binomial system associated to* the cell V.

Lemma 1.1 ([3]) *A binomial system associated to an alternating mixed cell V has exactly one real root in $(R_+)^n$.*

Let

$$V_1 = v_{1,1} + \cdots + v_{n,1}, \quad \cdots \quad , V_k = v_{1,k} + \cdots + v_{n,k}$$

be all the mixed cells of the subdivision τ_ω and for $i = 1, ..., n$ and $j = 1, ..., k$, let $a_{i,j}$ and $b_{i,j}$ be the endpoints of $v_{i,j}$. For $1 \le j \le n$, denote by B_j the binomial system

$$\begin{aligned} \alpha_{1,j} x^{a_{1,j}} + \beta_{1,j} x^{b_{1,j}} &= 0 \\ &\vdots \\ \alpha_{n,j} x^{a_{n,j}} + \beta_{n,j} x^{b_{n,j}} &= 0 \end{aligned}$$

where $\alpha_{i,j}$ and $\beta_{i,j}$ are the coefficients of the monomials $x^{a_{i,j}}$ and $x^{b_{i,j}}$ of p_i. Note that $sign(\alpha_{i,j}) = sign(a_{i,j})$ and $sign(\beta_{i,j}) = sign(b_{i,j})$.

The following theorem serves as a base for the Itenberg-Roy conjecture,

Theorem 1.2 ([3]) *For the polynomial system:*

$$p_i(x) = \sum_{q \in A_i} \alpha_{i,q} x^q \qquad i = 1, ..., n$$

consider the polynomial systems $H(x,t) = (h_1(x,t), ..., h_n(x,t))$ *where*

$$h_i(x,t) = \sum_{q \in A_i} \alpha_{i,q} x^q t^{\omega_i(q)} \qquad i = 1, ..., n \tag{2}$$

and $\omega_i : A_i \longrightarrow R$. *If t is positive and sufficiently small, then the number of isolated zeros of H(x,t) in* $(R_+)^n$ *is the total number of alternating mixed cells of the subdivision* τ_ω.

The Itenberg-Roy conjecture mainly suggests that the combinatorial construction of Theorem 1.2 yields an upper bound for *all* choices of coefficients, not just those appearing in the limits of a toric deformation. Notice that for each $1 \le i \le n$, $h_i(x,1) = p_i(x)$. After proper coordinate changes followed by multiplication of the polynomials by positive numbers on $H(x,t)$ in (2) [2, 3], which keeps the solution curves $x(t)$ of $H(x,t) = 0$ invariant at $t = 1$, the validity of the Itenberg-Roy conjecture would imply that the solution curve $x(t)$ with $x(1) \in (R_+)^n$ must be initiated from a solution of the binomial system associated to an alternating mixed cell of τ_ω in $(R_+)^n$ at $t = 0$. However, our counter example in (1) shows that some of those solution curves with $x(1) \in (R_+)^n$ may come from sources other than solutions of binomial systems associated to alternating mixed cells of τ_ω in $(R_+)^n$ at $t = 0$. To be more precise, with supports $A_1 = \{(0,0),(1,0),(0,1)\}$ and $A_2 = \{(0,0),(3,0),(0,3),(3,3)\}$ and lifting functions $\omega_1 : A_1 \longrightarrow R$ and $\omega_2 : A_2 \longrightarrow R$ where $\omega_1(1,0) = 1$, $\omega_1(0,0) = \omega_1(0,1) = 0$ and $\omega_2(3,3) = 2$, $\omega_2(0,0) = 1$, $\omega_2(0,3) = \omega_2(3,0) = 0$, our system in (1) becomes,

$$\begin{aligned} h_1(x_1,x_2,t) &= x_2 - x_1 t - 1 &= 0 \\ h_2(x_1,x_2,t) &= x_2^3 x_1^3 t^2 + 100x_2^3 - 900x_1^3 - 200t &= 0. \end{aligned} \tag{3}$$

Write $H(x_1, x_2, t) = (h_1(x_1, x_2, t), h_2(x_1, x_2, t))$. As we mentioned before, system in (3) at $t = 1$ which coincides with our system in (1) has three isolated zeros $a = (0.3177, 1.3177)$, $b = (0.6600, 1.6600)$, $c = (8.1206, 9.1206)$ in $(R_+)^2$. By decreasing t, one of these three branches of solution curves of $H(x_1, x_2, t) = 0$ emanated from a, b, c at $t = 1$ intersects $x_1 = 0$ at $t = \frac{1}{2}$ with $x_2 = 1$. Apparently, this branch, unlike the other two branches, does not stay in $(R_+)^3$ for all $0 \le t \le 1$. See figure 1.

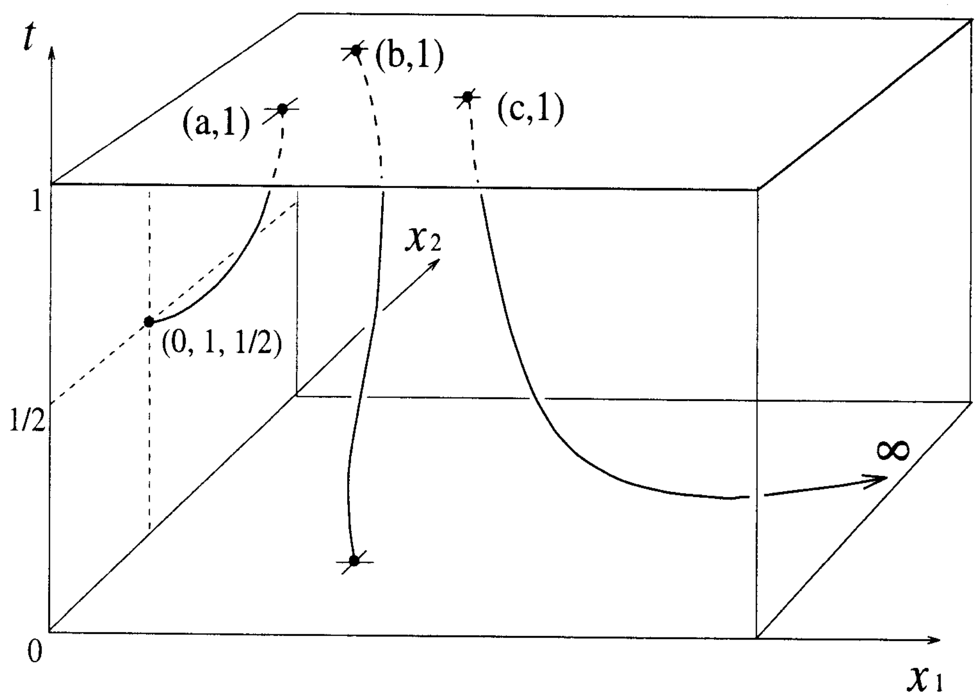

Figure 1: The solution curves of $H(x, t) = 0$ in $(R_+)^3$.

This observation initiates our project for further investigation on this problem. It appears that for appropriate lifting functions on the supports of a given real polynomial system $P(x) = (p_1(x), ..., p_n(x))$ where $x = (x_1, ..., x_n)$, the number of isolated solutions of $P(x) = 0$ in $(R_+)^n$ is less than or equal to the sum of the number of solution curves of the lifted system $H(x, t) = 0$ in $(R_+)^{n+1}$ near $t = 0$ as well as solution curves near $x_1 = 0, ..., x_n = 0$. This root count near $x_j = 0$ for $1 \le j \le n$ can be converted to combinatorial constructions as it was done for root count as $t \longrightarrow 0$. Namely, we just consider t as one additional variable, say $t = x_{n+1}$, and lifted system $H(x,t)$ with generic lifting functions ω_i on the support A_i of p_i may well be considered as a lifted system with the role of x_{n+1} $(= t)$ replaced by x_j for $1 \leq j \leq n$ and with lifting function ω_i^j on the *support*

$$A_i^j = \{q^j = (q_1, ..., q_{j-1}, q_{j+1}, ..., q_n, \omega_i(q)) | q = (q_1, ..., q_n) \in A_i\}$$

where $\omega_i^j(q^j) = q_j$. And, with $\Delta_i^j = conv(A_i^j), \Delta_M^j = \Delta_1^j + \cdots + \Delta_n^j$
and

$$A^j = \{a \in \Delta_M^j | a = a_1 + \cdots + a_n \text{ where } a_i \in A_i^j\},$$

the function $\omega^j : A^j \longrightarrow R$ with

$$\omega^j(a) = min\{\omega_1^j(a) + \cdots + \omega_n^j(a) | a = a_1 + \cdots + a_n\}$$

define a polyhedra subdivision τ_{ω^j} of Δ_M^j. From this point of view, we must notice that A^j's are no longer integer points and those functions ω_i^j are not generic, so the subdivision τ_{ω^j} they produce on Δ_M^j may not be a mixed subdivision. However, by *recursive* lifting on the cells in τ_{ω^j} that are not mixed, one may still be able to obtain the mixed cells hiding in the unmixed cells in τ_{ω^j}. As before, we may equip A_i^j with distribution of signs δ_i^j and determine the *alternating* mixed cells in τ_{ω^j} and the number of solution curves of $H(x,t) = 0$ in $(R_+)^{n+1}$ near $x_j = 0$ should be the number of alternating mixed cells in τ_{ω^j}. If we revise the definition of combinatorial upper bound for the system $P(x) = (p_1(x), ..., p_n(x))$ to be the maximum number of alternating mixed cells in all subdivisions τ_{ω^j} of Δ_m^j for $j = 1, ..., n$ as well as the subdivision τ_ω of Δ_M, then we believe that this combinatorial upper bound can serve as an upper bound for the number of isolated zeros of $P(x)$ in $(R_+)^n$.

The main topic of our project is to fulfill the theoretical justifications of the procedure we outlined above. While some of the suggested approaches may well change when the work progresses, the optimal goal of our project is to achieve a Multivariate Descartes' Rule based on the modification of the original suggestion made by I. Itenberg and M.-F. Roy.

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Publications Closely Related to the Project:

1. Find Isolated Zeros of Polynomial Systems in C^n with Stable Mixed Volumes (with T. Y. Li and T. Gao). Submitted
2. On Multivariate Descartes' Rule - a Counter example(with T. Y. Li), To appear: Beitrage zur Algebra und Geometrie, 39 (1998).
3. Counter examples to the connectivity conjecture of the mixed cells (with T. Y. Li). To appear: Journal of Discrete Comput. Geom..
4. The BKK Root Count in C^n (with T. Y. Li). Math. Comp. Vol. 65, No 216 (1996), 1477-1484.
5. Random product Start Systems using Homotopy (with T. Y. Li and T. Wang). Lectures in Applied Mathematics. Vol 32, pp. 503--512 (1996).

Other publications:

1. Higher Order Turning Points (with T. Y. Li). Journal of Applied Mathematics and Computation. Vol 64, 155-166 (1994).
2. Solving Real Polynomial Systems with Real Homotopies (with T. Y. Li). Math. Comp. Vol 60, No 202,669--680 (1993).
3. Nonlinear Homotopies for Solving deficient Polynomial System with Parameters (with T. Y. Li). SIAM J. Numer. Anal. Vol 29, No 4, 1104--1118 (1992).
4. Solving Deficient Polynomial System with Homotopies which Keep the Subschemes at Infinity Invariant (with T. Y. Li). Math. Comp. Vol 56, No 194 (1991), 693-710.
5. Homotopies for Solving the Kinematics of the Most General Six-And-Five-Degree of Freedom Manipulators (with T. Y. Li). Proc. of ASME on Mechanisms, DL -Vol 25, 249-252 (1990).

Past and current Collaboprators

BIOGRAPHICAL SKETCH

(This is a continuation page)

T. Gao, graduate student, Michigan State University.
Professor T. Y. Li, Michigan State University.
Dr. T. Wang, C-Mold/ Advanced CAE Technology.

Thesis advisor: Professor T. Y. Li, Michigan State University.

SUMMARY PROPOSAL BUDGET

YEAR 1

FOR NSF USE ONLY		
PROPOSAL NO.	DURATION (months)	
	Proposed	Granted
AWARD NO.		

ORGANIZATION
University of Central Arkansas

PRINCIPAL INVESTIGATOR / PROJECT DIRECTOR
Xiaoshen Wang

A. SENIOR PERSONNEL: PI/PD, Co-PI's, Faculty and Other Senior Associates (List each separately with title, A.7. show number in brackets)	NSF Funded Person-mos. CAL	ACAD	SUMR	Funds Requested By proposer	Funds granted by NSF (if different)
1. Xiaoshen Wang - Associate Professor	0.00	0.00	2.00	$ 9,052	$
2.					
3.					
4.					
5.					
6. (0) OTHERS (LIST INDIVIDUALLY ON BUDGET JUSTIFICATION PAGE)	0.00	0.00	0.00	0	
7. (1) TOTAL SENIOR PERSONNEL (1 - 6)	0.00	0.00	2.00	9,052	
B. OTHER PERSONNEL (SHOW NUMBERS IN BRACKETS)					
1. (0) POST DOCTORAL ASSOCIATES	0.00	0.00	0.00	0	
2. (0) OTHER PROFESSIONALS (TECHNICIAN, PROGRAMMER, ETC.)	0.00	0.00	0.00	0	
3. (0) GRADUATE STUDENTS				0	
4. (1) UNDERGRADUATE STUDENTS				700	
5. (0) SECRETARIAL - CLERICAL (IF CHARGED DIRECTLY)				0	
6. (0) OTHER				0	
TOTAL SALARIES AND WAGES (A + B)				9,752	
C. FRINGE BENEFITS (IF CHARGED AS DIRECT COSTS)				2,333	
TOTAL SALARIES, WAGES AND FRINGE BENEFITS (A + B + C)				12,085	
D. EQUIPMENT (LIST ITEM AND DOLLAR AMOUNT FOR EACH ITEM EXCEEDING $5,000.)					
TOTAL EQUIPMENT				0	
E. TRAVEL 1. DOMESTIC (INCL. CANADA AND U.S. POSSESSIONS)				1,255	
2. FOREIGN				2,910	
F. PARTICIPANT SUPPORT COSTS					
1. STIPENDS $ 0					
2. TRAVEL 0					
3. SUBSISTENCE 0					
4. OTHER 0					
(0) TOTAL PARTICIPANT COSTS				0	
G. OTHER DIRECT COSTS					
1. MATERIALS AND SUPPLIES				750	
2. PUBLICATION COSTS/DOCUMENTATION/DISSEMINATION				750	
3. CONSULTANT SERVICES				0	
4. COMPUTER SERVICES				0	
5. SUBAWARDS				0	
6. OTHER				0	
TOTAL OTHER DIRECT COSTS				1,500	
H. TOTAL DIRECT COSTS (A THROUGH G)				17,750	
I. INDIRECT COSTS (F&A)(SPECIFY RATE AND BASE) 42% of $9,752 (Rate: 42.00, Base: 9752)					
TOTAL INDIRECT COSTS (F&A)				4,095	
J. TOTAL DIRECT AND INDIRECT COSTS (H + I)				21,845	
K. RESIDUAL FUNDS (IF FOR FURTHER SUPPORT OF CURRENT PROJECTS SEE GPG II.D.7.j.)				0	
L. AMOUNT OF THIS REQUEST (J) OR (J MINUS K)				$ 21,845	$

M. COST SHARING PROPOSED LEVEL $ 0 — AGREED LEVEL IF DIFFERENT $

PI / PD TYPED NAME & SIGNATURE*	DATE	FOR NSF USE ONLY		
Xiaoshen Wang		INDIRECT COST RATE VERIFICATION		
ORG. REP. TYPED NAME & SIGNATURE*	DATE	Date Checked	Date Of Rate Sheet	Initials - ORG

SUMMARY PROPOSAL BUDGET COMMENTS - Year 1

** B-4 Undergraduate Students
The student will be used to write some programs and perform numerical experiments.
100 hous X $7 = $700
** C- Fringe Benefits
25% of $9052 for PI =$2,263
10% of $700 for student =$70
** E-1 Domestic Travel
one meeting.
Airfare: =$500
Room: 3 X $125=$375
Meal: 3 X $40 =$120
Registration = $200
Taxi = $60
Total =$1,255

** E-2 Foreign Travel
one meeting in Germany.
Airfare: = $1,500
Room: 5 X $140 =$700
Meal: 5X $50 =$250
Registration: = $400
Taxi: = $60
Total: = $2,910

** E- Travel
The travels are intended to present my results and exchange ideas at meetings.
1. Domestic travel
(one meeting) :
Airfare = $500
Room 3 x $125 = $375
Meal 3 x $ 40 = $120
Registration = $200
Taxi = $60
Total =$1,255
2. International Travel
(One meeting in Germany)
Airfare =$ 1,500
Room 5 x $140 = $700
Meal 5 x $ 50 = 250
Registration = $400
Taxi = $60
Total = $2,910

** G-1 Materials and Supplies

SUMMARY PROPOSAL BUDGET COMMENTS - Year 1

Computer software upgrade and other supplies. Total: $750

SUMMARY PROPOSAL BUDGET EXPLANATION - Year 1

Detailed Budget

A. Summer salary for PI: (2 summers) 2/9 x $40,732 = $9,052

B. 4. Undergraduate Student wage: 100 hours x $7/hour = $700

The student will be used to write some programs and perform numerical experiments.
Total salaries and wages: = $9,752
C. Fringe benefit
PI : 25% of $9,052 = $2,263
Undergraduate student: 10% of $700 = $70
Total fringe benefit : = $2,333
Total salaries, wages and fringe benefits = $12,085
D. Total equipment: = $0
E. I intend to attend meetings to present the results.
1. Domestic travel (one meeting) :
Airfare = $500
Room 3 x $125 = $375
Meal 3 x $ 40 = $120
Registration = $200
Taxi = $60
Total =$1,255
2. International Travel (One meeting in Germany)
Airfare = $1,500
Room 5 x $140 = $700
Meal 5 x $ 50 = $250
Registration = $400
Taxi = $60
Total = $2,910
F. Participant Support Costs = $0
G. Other Direct Cost
1. Materials and Supplies
Computer software upgrading and other supplies =$750
2. Publication Costs/Documentation/Dissemination :=$750
Total other direct cost: =$1,500
H. Total Direct Cost: =$17,750
I. Indirect Costs (F & A) : 42% of $9,752 =$4,095
J. Total Direct and Indirect Cost: =$21,845
K. Residual Funds: =$0
L. Amount of this request =$21,845

SUMMARY PROPOSAL BUDGET

YEAR 2

ORGANIZATION	FOR NSF USE ONLY		
University of Central Arkansas	PROPOSAL NO.	DURATION (months)	
		Proposed	Granted
PRINCIPAL INVESTIGATOR / PROJECT DIRECTOR Xiaoshen Wang	AWARD NO.		

A. SENIOR PERSONNEL: PI/PD, Co-PI's, Faculty and Other Senior Associates (List each separately with title, A.7. show number in brackets)	NSF Funded Person-mos. CAL	ACAD	SUMR	Funds Requested By proposer	Funds granted by NSF (if different)
1. Xiaoshen Wang - Associate Professor	0.00	0.00	2.00	$ 9,052	$
2.					
3.					
4.					
5.					
6. (0) OTHERS (LIST INDIVIDUALLY ON BUDGET JUSTIFICATION PAGE)	0.00	0.00	0.00	0	
7. (1) TOTAL SENIOR PERSONNEL (1 - 6)	0.00	0.00	2.00	9,052	
B. OTHER PERSONNEL (SHOW NUMBERS IN BRACKETS)					
1. (0) POST DOCTORAL ASSOCIATES	0.00	0.00	0.00	0	
2. (0) OTHER PROFESSIONALS (TECHNICIAN, PROGRAMMER, ETC.)	0.00	0.00	0.00	0	
3. (0) GRADUATE STUDENTS				0	
4. (1) UNDERGRADUATE STUDENTS				700	
5. (0) SECRETARIAL - CLERICAL (IF CHARGED DIRECTLY)				0	
6. (0) OTHER				0	
TOTAL SALARIES AND WAGES (A + B)				9,752	
C. FRINGE BENEFITS (IF CHARGED AS DIRECT COSTS)				2,333	
TOTAL SALARIES, WAGES AND FRINGE BENEFITS (A + B + C)				12,085	
D. EQUIPMENT (LIST ITEM AND DOLLAR AMOUNT FOR EACH ITEM EXCEEDING $5,000.)					
TOTAL EQUIPMENT				0	
E. TRAVEL 1. DOMESTIC (INCL. CANADA AND U.S. POSSESSIONS)				1,255	
2. FOREIGN				0	
F. PARTICIPANT SUPPORT COSTS 1. STIPENDS $ 0 2. TRAVEL 0 3. SUBSISTENCE 0 4. OTHER 0					
(0) TOTAL PARTICIPANT COSTS				0	
G. OTHER DIRECT COSTS					
1. MATERIALS AND SUPPLIES				750	
2. PUBLICATION COSTS/DOCUMENTATION/DISSEMINATION				750	
3. CONSULTANT SERVICES				0	
4. COMPUTER SERVICES				0	
5. SUBAWARDS				0	
6. OTHER				0	
TOTAL OTHER DIRECT COSTS				1,500	
H. TOTAL DIRECT COSTS (A THROUGH G)				14,840	
I. INDIRECT COSTS (F&A)(SPECIFY RATE AND BASE) % salaries & wages (Rate: 42.00, Base: 9752)					
TOTAL INDIRECT COSTS (F&A)				4,095	
J. TOTAL DIRECT AND INDIRECT COSTS (H + I)				18,935	
K. RESIDUAL FUNDS (IF FOR FURTHER SUPPORT OF CURRENT PROJECTS SEE GPG II.D.7.j.)				0	
L. AMOUNT OF THIS REQUEST (J) OR (J MINUS K)				$ 18,935	$

M. COST SHARING PROPOSED LEVEL $ 0	AGREED LEVEL IF DIFFERENT $		
PI / PD TYPED NAME & SIGNATURE* Xiaoshen Wang	DATE	FOR NSF USE ONLY INDIRECT COST RATE VERIFICATION	
ORG. REP. TYPED NAME & SIGNATURE*	DATE	Date Checked / Date Of Rate Sheet	Initials - ORG

SUMMARY PROPOSAL BUDGET COMMENTS - Year 2

** E- Travel

I intend to attend meetings to present the results.

1. Domestic travel (one meeting) :

Airfare		= $500
Room	3 x $125	= $375
Meal	3 x $ 40	= $120
Registration		= $200
Taxi		= $60
Total		=$1,255

2. International Travel

Total	= $

SUMMARY PROPOSAL BUDGET EXPLANATION - Year 2

Detailed Budget

A. Summer salary for PI: (2 summers) 2/9 x $40,732 = $9,052

B. 4. Undergraduate Student wage: 100 hours x $7/hour = $700

The student will be used to write some programs and perform numerical experiments.
Total salaries and wages: = $9,752
C. Fringe benefit
PI : 25% of $9,052 = $2,263
Undergraduate student: 10% of $700 = $70
Total fringe benefit : = $2,333
Total salaries, wages and fringe benefits = $12,085
D. Total equipment: = $
E. I intend to attend meetings to present the results.
1. Domestic travel (one meeting) :
Airfare = $500
Room 3 x $125 = $375
Meal 3 x $ 40 = $120
Registration = $200
Taxi = $60
Total =$1,255
2. International Travel
Total = $0
F. Participant Support Costs = $0
G. Other Direct Cost
1. Materials and Supplies
Computer software upgrading and other supplies =$750
2. Publication Costs/Documentation/Dissemination :=$750
Total other direct cost: =$1,500
H. Total Direct Cost: =$14,840
I. Indirect Costs (F & A) : 42% of $9,752 =$4,095
J. Total Direct and Indirect Cost: =$18,935
K. Residual Funds: =$0
L. Amount of this request =$18,935

SUMMARY PROPOSAL BUDGET

Cumulative

ORGANIZATION: University of Central Arkansas	FOR NSF USE ONLY — PROPOSAL NO. / DURATION (months): Proposed / Granted
PRINCIPAL INVESTIGATOR / PROJECT DIRECTOR: Xiaoshen Wang	AWARD NO.

A. SENIOR PERSONNEL: PI/PD, Co-PI's, Faculty and Other Senior Associates (List each separately with title, A.7. show number in brackets)	NSF Funded Person-mos. CAL	ACAD	SUMR	Funds Requested By proposer	Funds granted by NSF (if different)
1. Xiaoshen Wang - Associate Professor	0.00	0.00	4.00	$ 18,104	$
2.					
3.					
4.					
5.					
6. ( ) OTHERS (LIST INDIVIDUALLY ON BUDGET JUSTIFICATION PAGE)	0.00	0.00	0.00	0	
7. (1) TOTAL SENIOR PERSONNEL (1 - 6)	0.00	0.00	4.00	18,104	
B. OTHER PERSONNEL (SHOW NUMBERS IN BRACKETS)					
1. (0) POST DOCTORAL ASSOCIATES	0.00	0.00	0.00	0	
2. (0) OTHER PROFESSIONALS (TECHNICIAN, PROGRAMMER, ETC.)	0.00	0.00	0.00	0	
3. (0) GRADUATE STUDENTS				0	
4. (2) UNDERGRADUATE STUDENTS				1,400	
5. (0) SECRETARIAL - CLERICAL (IF CHARGED DIRECTLY)				0	
6. (0) OTHER				0	
TOTAL SALARIES AND WAGES (A + B)				19,504	
C. FRINGE BENEFITS (IF CHARGED AS DIRECT COSTS)				4,666	
TOTAL SALARIES, WAGES AND FRINGE BENEFITS (A + B + C)				24,170	
D. EQUIPMENT (LIST ITEM AND DOLLAR AMOUNT FOR EACH ITEM EXCEEDING $5,000.)					
TOTAL EQUIPMENT				0	
E. TRAVEL 1. DOMESTIC (INCL. CANADA AND U.S. POSSESSIONS)				2,510	
2. FOREIGN				2,910	
F. PARTICIPANT SUPPORT COSTS					
1. STIPENDS $ 0					
2. TRAVEL 0					
3. SUBSISTENCE 0					
4. OTHER 0					
(0) TOTAL PARTICIPANT COSTS				0	
G. OTHER DIRECT COSTS					
1. MATERIALS AND SUPPLIES				1,500	
2. PUBLICATION COSTS/DOCUMENTATION/DISSEMINATION				1,500	
3. CONSULTANT SERVICES				0	
4. COMPUTER SERVICES				0	
5. SUBAWARDS				0	
6. OTHER				0	
TOTAL OTHER DIRECT COSTS				3,000	
H. TOTAL DIRECT COSTS (A THROUGH G)				32,590	
I. INDIRECT COSTS (F&A)(SPECIFY RATE AND BASE)					
TOTAL INDIRECT COSTS (F&A)				8,191	
J. TOTAL DIRECT AND INDIRECT COSTS (H + I)				40,781	
K. RESIDUAL FUNDS (IF FOR FURTHER SUPPORT OF CURRENT PROJECTS SEE GPG II.D.7.j.)				0	
L. AMOUNT OF THIS REQUEST (J) OR (J MINUS K)				$ 40,781	$

M. COST SHARING PROPOSED LEVEL $ 0	AGREED LEVEL IF DIFFERENT $	
PI / PD TYPED NAME & SIGNATURE*: Xiaoshen Wang	DATE	FOR NSF USE ONLY — INDIRECT COST RATE VERIFICATION
ORG. REP. TYPED NAME & SIGNATURE*	DATE	Date Checked / Date Of Rate Sheet / Initials - ORG

NSF Form 1030 (10/97) Supersedes all previous editions

C*SIGNATURES REQUIRED ONLY FOR REVISED BUDGET (GPG III.B)

Current and Pending Support

(See GPG Section II.D.8 for guidance on information to include on this form.)

The following information should be provided for each investigator and other senior personnel. Failure to provide this information may delay consideration ofthis proposal.

Investigator: **Xiaoshen Wang**

Other agencies (including NSF) to which this proposal has been/will be submitted.

Support: ☐ Current ☒ Pending ☐ Submission Planned in Near Future ☐ *Transfer of Support

Project/Proposal Title: **Multivariate Descartes' Rule of Signs (This is the proposal being submitted.)**

Source of Support: **NSF**

Award Amount (or Annual Rate): $ **40,780** Period Covered: **06/01/98 - 05/31/00**

Location of Project: **University of Central Arkansas**

Person-Months Committed to the Project. Cal: Acad: Summ: **2.00**

Support: ☐ Current ☐ Pending ☐ Submission Planned in Near Future ☐ *Transfer of Support

Project/Proposal Title:

Source of Support:

Award Amount (or Annual Rate): $ Period Covered:

Location of Project:

Person-Months Committed to the Project. Cal: Acad: Summ:

Support: ☐ Current ☐ Pending ☐ Submission Planned in Near Future ☐ *Transfer of Support

Project/Proposal Title:

Source of Support:

Award Amount (or Annual Rate): $ Period Covered:

Location of Project:

Person-Months Committed to the Project. Cal: Acad: Summ:

Support: ☐ Current ☐ Pending ☐ Submission Planned in Near Future ☐ *Transfer of Support

Project/Proposal Title:

Source of Support:

Award Amount (or Annual Rate): $ Period Covered:

Location of Project:

Person-Months Committed to the Project. Cal: Acad: Summ:

Support: ☐ Current ☐ Pending ☐ Submission Planned in Near Future ☐ *Transfer of Support

Project/Proposal Title:

Source of Support:

Award Amount (or Annual Rate): $ Period Covered:

Location of Project:

Person-Months Committed to the Project. Cal: Acad: Summ:

*If this project has previously been funded by another agency, please list and furnish information for immediately preceding funding period.

FACILITIES, EQUIPMENT & OTHER RESOURCES

FACILITIES: Identify the facilities to be used at each performance site listed and, as appropriate, indicate their capacities, pertinent capabilities, relative proximity, and extent of availability to the project. Use "Other" to describe the facilities at any other performance sites listed and at sites for field studies. USE additional pages as necessary.

Laboratory:

Clinical:

Animal:

Computer: **A computer with a Pentium 100 MHz processor loaded with many application software programs is in my office. It is sufficient for numerical experiments, word processing and data communication, which are the basic needs of the project.**

Office: **The office assigned to me by the department is Room 206E in Mail Hall at University of Central Arkansas. It is very convenient and has enough room for me to do research there.**

Other: ____________________

MAJOR EQUIPMENT: List the most important items available for this project and, as appropriate identifying the location and pertinent capabilities of each.

OTHER RESOURCES: Provide any information describing the other resources available for the project. Identify support services such as consultant, secretarial, machine shop, and electronics shop, and the extent to to which they will be available for the project. Include an explanation of any consortium/contractual arrangements with other organizations.