Annual Report for Period: 08/2003 - 08/2004 **Submitted on:** 02/25/2005
Principal Investigator: De Loera, Jesus . **Award ID:** 0309694
Organization: U of Cal Davis
Title:
Computational Polyhedral Geometry: Applications in Algebra, Combinatorics, and Optimization

Project Participants

Senior Personnel

Name: De Loera, Jesus
Worked for more than 160 Hours: Yes
Contribution to Project:

Post-doc

Graduate Student

Name: Yoshida, Ruriko
Worked for more than 160 Hours: Yes
Contribution to Project:
She was a graduate student writing a Ph.D thesis on a topic part of the grant.

Name: Ahmed, Maya
Worked for more than 160 Hours: Yes
Contribution to Project:
She was a graduate student writing a Ph.D thesis on a topic part of the grant.

Name: McAllister, Tyrrell
Worked for more than 160 Hours: Yes
Contribution to Project:
He is a graduate student, still working toward his Ph.D. His thesis topic has been essentially one of the parts of the grant.

Undergraduate Student

Name: Huggins, Peter
Worked for more than 160 Hours: Yes
Contribution to Project:
Undergraduate student that wrote a senior thesis and helped developed the software final product of this grant.

Name: Haws, David
Worked for more than 160 Hours: Yes
Contribution to Project:
Undergraduate student who did research and was also a programmer for the software development that is supported in this grant.

Name: Tauzer, Jeremy
Worked for more than 160 Hours: Yes
Contribution to Project:
Undergraduate student who was a programmer for the software development

that has been part of this grant.

Technician, Programmer

Other Participant

Research Experience for Undergraduates

Organizational Partners

Other Collaborators or Contacts

Co-authors of some of the work produced under support of this grant:

1) Bernd Sturmfels (UC Berkeley)
2) Matthias Beck (SF state Univ)
3) Michael Develin (AIM)
4) Julian Pfeifle (Univ. Barcelona, Spain)
5) Richard Stanley (MIT)
6) Shmuel Onn (Technion Haifa, Israel)
7) Lewis Bowen (Indiana Univ)
8) Francisco Santos (Univ. of Cantabria Spain)
9) Welleda Baldoni-Silva (Univ of Rome, Italy)
10) Michele Vergne (Ecole Polytechnique Paris, France)
11) Raymond Hemmecke (Univ. Magdeburg Germany)

Activities and Findings

Research and Education Activities:

Our topic of research is on the general area of Computational Combinatorial mathematics, attacking problems in computational geometry, computational algebra, and optimization. The two main themes have been:

0) The investigation of algebraic and combinatorial properties of the lattice points that lie inside a convex body, in particular polyhedra.

1) Applications of (0) to computer algebra, in particular computational representation theory and commutative algebra.

2) The design of fast algorithms to solve problems about lattice points (e.g. count how many are there, find an optimal one, etc).

3) Software implementation of (2).

Of course, three graduate students have based their theses on this project and three undergraduates have been introduced to research through this project. So the educational component of the project is very strong.

Findings:

Under this grant, my students and I have developed succesfully both theory and software:

1) On the theoretical side, noteworthy results were

a) In joint work with my student T. McAllister we disproved

a well-known ten-year-old conjecture by Berenstein and Kirillov on the Representation theory of Lie Algebras (paper has now appeared in print). We use our polytope techniques to achieve this.
It is worth mentioning that some referees of the grant proposal expressed skepticism that we would achieve this goal, one of them said it was impossible. We are more proud to have reached our high goal. One of the referee's of our paper stated our techniques could have important impact in Computational Algebra (i.e. Representation theoretic aspects).

b) Out of a pool of 132 papers TWO papers we submitted were accepted to appear in the prestigious conference IPCO X in New York city. Only 35 papers were accepted altogether.
The first paper is a report on the development of new, non-traditional algorithms for integer programming. We explained theory and presented experimental results (see part 2 of software).

The second paper answers another problem we stated in this grant proposal. The paper shows how one can in polynomial time transform any linear or integer programming problem into a form that uses only 0,1 coefficients in the variables. As a consequence the family of multi-way transportation optimization problems is proved to contain ALL possible integer programming problems.

2) A new final version of our software LattE (available online at www.math.ucdavis.edu/~latte) was released. It includes now the ability to solve integer linear programs and produce Ehrhart quasipolynomials. We receive weekly requests to use our software.

Training and Development:

For the graduate students involved the projects mentioned in this grant are the source of research problems for their dissertation. The undergraduates learned about asking research questions and working in teams toward a goal. All of the students had to give oral presentations or posters explaining their research in front of an audience.

Outreach Activities:

One of the ``popular'' applications of the mathematics we do is to count magic squares (these are n by n tables of integer numbers whose row sums, column sums and two diagonal sums are all equal). These puzzles are part of the popular culture and lots of people (non-mathematicians) play with them. I gave several lectures for young students (from K-12 to college) about them and the ``hard'' math behind them. One of my Ph.D Students and I were cited in the magazine Science News (Vol 189, issue 2426-December 20 2003).
Our research was also featured in the magazine ``{UC Davis Science News}'' (June 16 2004).

I was Faculty advisor of the \emph{ UC Davis Math Club}. I enjoyed sharing with the undegraduate students exciting cultural and educational activities. I helped them organized activities such as ``career orientation'', math exhibit at Explorit's science museum, math movie night

Journal Publications

J.A. De Loera, D.Haws, R. Hemmecke, P. Huggins, B. Sturmfels, R. Yoshida, "Short rational functions for toric algebra and applications", Journal of Symbolic Computation, p. 959, vol. 38, (2004). Published

J.A. De Loera, D.Haws, R. Hemmecke, P. Huggins, R. Yoshida, "Three kinds of Integer Programming Algorithms based on Barvinok's rational functions", n proceedings {\em Tenth International Conference
in Integer Programming and Combinatorial Optimization, New York NY, June 2004}
Lecture Notes in Computer Science, p. 244, vol. 3064, (2004). Published

J.A. De Loera and S. Onn, "All rational polytopes are transportation polytopes and all polytopal integer sets are contingency tables", n proceedings {\em Tenth International Conference in Integer Programming and
Combinatorial Optimization, New York NY, June 2004} Lecture Notes in Computer Science, p. 338, vol. 3064, (2004). Published

J.A. De Loera, R. Hemmecke, J. Tauzer, and R. Yoshida, "Effective lattice point counting in rational convex polytopes", Journal of Symbolic Computation, p. 277, vol. 38, (2004). Published

J.A. De Loera and T.B McAllister, "Vertices of Gelfand-Tsetlin polytopes", Discrete and Computational Geometry, p. 459, vol. 32, (2004). Published

L. Bowen, J.A. De Loera, M. Develin and F. Santos, "The Gromov norm of the product of two surfaces", Topology, p. , vol. , ( ). Accepted

W. Baldoni-Silva, J.A De Loera, and M. Vergne, "Counting integer flows in networks", Foundations of Computational Mathematics, p. 277, vol. 4, (2004). Published

J.A De Loera and S. Onn, "Markov bases of 3-way tables
are arbitrarily complicated", Journal of symbolic computation, p. , vol. , ( ). Submitted

J.A. De Loera, "The many aspects of counting lattice points in polyhedra", Mathematische Semester Berichte, p. , vol. , ( ). Submitted

Books or Other One-time Publications

Web/Internet Site

URL(s):

www.math.ucdavis.edu/~latte

Description:

In the award we promised to developed certain algorithms
that now have been implemented as part of this software
producing faster performance.

Other Specific Products

Product Type:

Software (or netware)

Product Description:

The software LattE is allows mathematicians to answer
questions about the lattice points inside a polyhedron.

Sharing Information:

It is freely distributed over the internet (see web site).

Contributions

Contributions within Discipline:

Within computational mathematics, in particular the areas of computational geometry and computer algebra, we have designed new algorithms and created useful software. That this techniques have impact is evident that new software is being produced using the same ideas. Our papers were selected to one of the best conferences in the field (IPCO).

Besides of what was already mentioned in the findings section. I would like to say that our new algorithm to compute the Grobner bases of a toric ideal has also made impact in computer algebra. I was recently been invited as plenary speaker in a conference devoted to the practical effectiveness of Grobner bases (Grobner bases have many applications, e.g. the solution of polynomial systems of equations).

Contributions to Other Disciplines:

As I mentioned in the finding section our results on the vertices of Gelfand-Tsetlin polytopes has attracted alot of attention. There is the expectation that computational representation theory will benefit with better faster geometric algorithms.

The use of rational functions in the calculations to count lattice points has influence in a wide variety of areas. From the most abstract (e.g. algebraic geometry, toric varieties) to the most applied (Statistics, Optimization). Our software and theoretical results have attracted alot of attention from people in these wide range from areas and even industry because counting lattice points in convex bodies is related to cryptograpy.

Finally, in compiler design, computer scientist need to count lattice points in order to estimate the number of times a computer instruction will be executed or the amount of memory that will be allocated during a subroutine call. We are proud to say that our work has been not only quoted in this context but a new software package was developed based on LattE.

Contributions to Human Resource Development:

I am proud to report 2 of the graduate students have finished their Ph.D work (Maya Ahmed and Ruriko Yoshida). Dr. Ahmed is working for a start-up software company while Dr. Yoshida is a postdoc at Duke University. My undergraduate students were recently placed into graduate school (Huggins Berkeley, Haws Davis).

Contributions to Resources for Research and Education:

Our software LattE has definitly contributed as a resource for research. Several researchers recently published solutions to a fascinating conjecture about the formula of the Ehrhart polynomial of the cyclic polytope which was discovered using LattE. Statisticians have recently used LattE as a benchmark for other algorithms.

Contributions Beyond Science and Engineering:

Not applicable yet.

Special Requirements

Special reporting requirements: None

Change in Objectives or Scope: None

Unobligated funds: less than 20 percent of current funds

Animal, Human Subjects, Biohazards: None

Categories for which nothing is reported:

Organizational Partners

Any Book