Math and Computers, Math 165
review problems

1. Calculate the remainder of $x^3 - x^2y - x^2z + x$ by $f_1 = x^2y - z$, and $f_2 = xy - 1$. Compute the remainder of f on division by (f_1, f_2).

2. Using Lagrange multipliers find the point(s) on the surface $x^4+y^2+z^2-1 = 0$ closest to the point $(1, 1, 1)$. HINT: You have to solve a system of equations!

3. Using Gröbner bases find all critical points of the polynomial function (check your calculus book for the definition).

$$f(x, y) = (x^2 + y^2 - 4)(x^2 + y^2 - 1) + (x - \frac{3}{2})^2 + (y - \frac{3}{2})^2$$

4. What is the Buchberger criterion, show an example.

5. Find all solutions of the system of equations $x^2 + y^2 + z^2 - 1 = 0, x^2 + y^2 + z^2 - 2x = 0, 2x - 3y - z = 0$.

6. State the Hilbert Nullstellensatz.