## CALCULUS, Math 16C Homework 7 (review)

1. When predicting population growth, demographers must consider birth and death rates as well as the net change caused by the difference between rates of immigration and emigration. Let P be the population at time tand N be the net increase per unit time due to the difference between immigration and emigration. The rate of growth for the population is given by

$$\frac{dP}{dt} = kP + N$$

Solve this differential equation and find P as a function of time.

- 2. Find any critical points and relative extrema of the function  $x^3 3xy + y^2$ .
- 3. Using Lagrange Multipliers locate any extrema of the function

$$x^2 + y^2 + z^2$$

subject to x + z = 6 and y + z = 8.

- 4. The production function of a manufacturer is given by f(x, y) = 4x + xy + 2y. Assume that the total amount available for labor x and capital y is 2000 dollars and that units of labor and capital cost \$20 and \$4 respectively. Find the maximum production level for this manufacturer.
- 5. A jeweler wishes to make a rectangular pill box such that the top and bottom will be copper plated and the sides silver plated. If silver plating costs 6 dollars per  $cm^2$  and copper plating costs 2 dollars per  $cm^2$ , what are the dimensions of the least costly pill box having  $8/3 \ cm^3$  as its volume?
- 6. Use a double integral to find the area of the region whose upper bound is the function  $y = \sqrt{x+3}$  the lower boundary is  $y = \frac{x+3}{3}$ .
- 7. Which of the following series converge? When possible compute the value of the sum. a)  $\sum_{n=0}^{\infty} (1/3)^n$ , b)  $\sum_{n=0}^{\infty} 2(1/4)^n$  c)  $\sum_{n=1}^{\infty} \frac{4^n}{3^n+1}$  d)  $\sum_{n=1}^{\infty} (\frac{1}{n^2} \frac{1}{n^3})$ .
- 8. Find the radius of convergence for the power series

$$\sum_{n=1}^{\infty} \frac{(2x+1)^n}{n^2}$$

- 9. (a) Use the Maclaurin series of cos(x) to state the first three non-zero terms of the Maclaurin series for  $\frac{1-cos(x)}{x^2}$ .
  - (b) Explain how you would use your answer in (a) to estimate  $\int_0^1 \frac{1 \cos(x)}{x^2} dx$ .