Math 16C (De Loera)
Mid-term exam 2
May 28 2010

Name: SOLUTION
Student ID# ———
Section number ———

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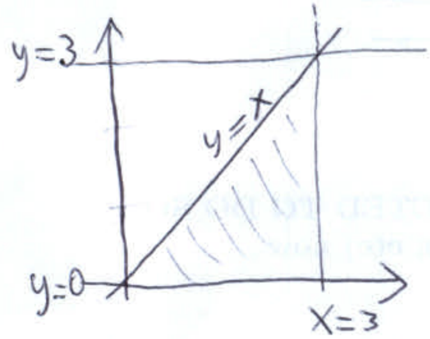
Show your work on every problem. Correct answers with no support work will not receive full credit. Be organized, clean, and use the notation appropiately. No calculators are allowed, nor is any assistance from classmates, notes, or books. You should only have a writing and an erasing implement on your desk.

Please write legibly!!

#	Student's Score	Maximum possible Score
1		7
2		8
3		8
4		7
Total points		30

1. (7 points from Homework 4, 7.8, 33:) Compute the integral $\int_0^3 \int_y^3 e^{x^2}\, dx dy$.



One needs to change the order of integration

$$\int_0^3 \int_0^x e^{x^2}\, dy\, dx$$

$$= \int_0^3 e^{x^2} y \Big|_0^x dx = \int_0^3 x e^{x^2} dx$$

$$= \frac{1}{2} e^{x^2} \Big|_0^3 = \frac{1}{2} e^9 - \frac{1}{2}$$

+1 if region is correct, +1 if you change order of integration

+1 for each correct limit of integration (total 2)

+1 if first integral was solved correctly

+1 if second integral was solved correctly

+1 if final evaluation is correct.

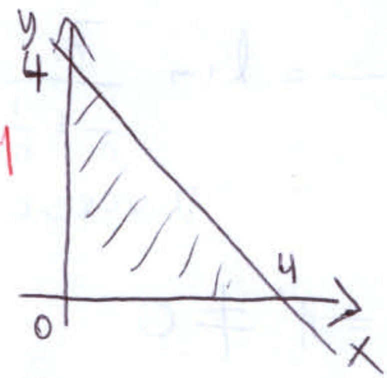
VALUE on points for each step shown in red

2. (8 points) Find the volume of the solid bounded by the graphs of the surfaces with equations $z = (xy)$, $z = 0$, $y = 0$, $x + y = 4$, and $x = 0$. Draw the planar region over which you are integrating.

+1 +1 +1

$$\int_0^4 \left(\int_0^{4-x} \underbrace{x y}_{+1} \, dy \right) dx$$

+1



Region of integration

$$= \int_0^4 \underbrace{\frac{x y^2}{2}}_{+1} \Big|_0^{4-x}$$

$$= \int_0^4 \frac{x(4-x)^2}{2} \, dx$$

$$= \frac{1}{2} \int_0^4 x\left(16 - 8x + x^2\right) dx$$

$$= \underbrace{\frac{1}{2} \int_0^4 \left(x^3 - 8x^2 + 16x\right) dx}_{+1} = \frac{1}{2}\left(\frac{x^4}{4} - \frac{8x^3}{3} + \frac{16}{2}x^2 \Big|_0^4 \right)$$

$$= \underbrace{\frac{4^4}{8} - \frac{8 \cdot 4^3}{6} + \frac{16 \cdot 4^2}{4}}_{+1} = \frac{4^3}{2} - \frac{4^4}{3} + 4^3$$

Point value of steps shown in red

3. (8 points) Which of the following sequences and series converge? Remember to justify your answer!

(a) $(-1/3)e^{-n}$

(b) $\frac{7n^3}{n^2+1}$

(c) $\sum_{n=0}^{\infty} \frac{n^2}{n(n+1)}$

(d) $\sum_{n=0}^{\infty}(\frac{n^4}{1-n^4} + \frac{1}{2^n})$

(e) $\sum_{n=0}^{\infty} 4(\frac{1}{4})^n$

a) $\lim_{n\to\infty} \left(-\frac{1}{3}\right)\frac{1}{e^n} = 0$ CONVERGES +1

b) $\lim_{n\to\infty} \frac{7n^3}{n^2+1} = \lim \frac{7}{\frac{1}{n}+\frac{1}{n^3}} = \infty$ +1

DIVERGES

c) $\lim_{n\to\infty} \frac{n^2}{n^2+n} = \lim_{n\to\infty} \frac{1}{1+\frac{1}{n}} = 1 \neq 0$

By n-th term test series is Divergent.

d) $\lim_{n\to\infty} \frac{n^4}{1-n^4} + \frac{1}{2^n} = \lim_{n\to\infty} \frac{1}{\frac{1}{n^4}-1} + \frac{1}{2^n} = -1 \neq 0$

n-th term test serias is Divergent

e) $\sum_{n=0}^{\infty} 4\left(\frac{1}{4}\right)^n = 4\sum_{n=0}^{\infty}\left(\frac{1}{4}\right)^n$ $\left|\frac{1}{4}\right| < 1$ by

Geometric serias test

Converges.

For parts c, d, e: 1 point for using the right test
1 point for applying correctly the test
(e.g computing the limit correctly)

4. (7 points)

(a) (Homework 6, 10.4, 29) Using Taylor's theorem find the first five terms of the power series centered at $c = 0$ for the function $f(x) = 1/(x^2 + 1)$.

(b) For which values of x does the power series $\sum_{n=0}^{\infty} \frac{(-1)^n (x-2)^n}{(n+1)^2}$ converge? ENOUGH to show INTERVAL OF CONVERGE, NO NEED TO CHECK EXTREMES

(a) The Taylor series at $c=0$ is

$$1 - x + x^2 - x^3 + x^4 - x^5 + \cdots + (-1)^n x^n + \cdots$$

and converges for $|x|<1$ thus Taylor for $\frac{1}{x^2+1}$

$$= 1 - x^2 + x^4 - x^6 + x^8 - x^{10} + \cdots + (-1)^n x^{2n} + \cdots$$

(b) Apply ratio test

$$\lim_{n\to\infty} \left| \frac{(-1)^{n+1}(x-2)^{n+1}}{(n+2)^2} \cdot \frac{(n+1)^2}{(-1)^n (x-2)^n} \right| = \lim_{n\to\infty} \left| (x-2) \frac{(n+1)^2}{(n+2)^2} \right|$$

$$= \lim_{n\to\infty} |x-2| \left| \frac{\left(1+\frac{1}{n}\right)^2}{\left(1+\frac{2}{n}\right)^2} \right| = |x-2| \leftarrow$$ This converges when $|x-2|<1$ (and possibly when $|x-2|=1$)

$$-1 < x-2 < 1 \implies \underline{1 < x < 3}$$

For part (a) There were two possible ways to solve the problem:

CASE 1: Using Taylor series of $\frac{1}{x+1}$

1 point for recognizing this, +1 point for writing series correctly, +1 for doing substitution correctly to get $\frac{1}{x^2+1}$, +1 point giving enough terms $\longrightarrow$

Case 2: 3 points for computing derivatives, +1 point for writing down the Taylor series using $\frac{f^{(n)}(c)}{n!}$ with $c=0$ for the coefficients.

Part (b) +1 for setting up ratio test

+1 for correct computation of limit

+1 for knowing power series converges when

$|x-2|<1$