

**CALCULUS, Math 17C**  
**Homework 2 Due April 20**

1. Read sections 10.3-10.6.
2. Solve exercises 10.3: 17, 21, 25,29, 33,35,39,41,45,47,49
3. Solve exercises 10.4: 1,7,10,17,20,25,26,27,29
4. Solve exercises 10.5: 17,25,29,31,35,39
5. Solve exercises 10.6: 1,3,5,9,12,26,27,29
6. Additional “in-depth” problems:
7. The Diffusion Equation:

*Diffusion* is the net movement of a substance (e.g., ions or molecules) from a region of high concentration to a region of low concentration. This is also referred to as the movement of a substance down a concentration gradient. A gradient is the change in the value of a quantity (e.g., concentration, pressure, temperature) with the change in another variable (e.g., distance). For example, a change in concentration over a distance is called a concentration gradient. Diffusion is an important mechanism for transport in many biological systems.

The concentration of diffusing particles  $c(x, t)$  in one spatial dimension obeys the *partial differential equation*

$$\frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial x^2},$$

where  $x$  is the position in space,  $t$  is time, and  $D$  is the “*diffusion constant*”, which depends of the substance that is diffusing and the substrate in which it is diffusing.

- (a) If  $c$  is in molecules/mm<sup>3</sup>,  $x$  is in mm, and  $t$  is in seconds, What are the units of  $D$ ?
- (b) Verify that the following function is a solution to the diffusion equation

$$c(x, t) = \frac{1}{\sqrt{4\pi Dt}} \exp\left(-\frac{x^2}{4Dt}\right), \quad t > 0.$$

- (c) The solution above is a Gaussian function (bell shaped curve) for fixed  $t$ . Describe what happens to the solution as time  $t$  increases.

8. Energy expenditure during locomotion:

The average energy  $E$  (in kcal) needed for a lizard to walk or run a distance of 1 km has been modeled by the equation

$$E(m, v) = 2.65 m^{2/3} + \frac{3.5 m^{3/4}}{v}$$

where  $m$  is the body mass of the lizard (in grams) and  $v$  is its speed (in km/hr). Find the linearization of the energy function at  $(m, v) = (400, 8)$ . (from C. Robbins, 1993).