CALCULUS, Math 17C
Homework 2 Due April 20

1. Read sections 10.3-10.6.
2. Solve exercises 10.3: 17, 21, 25, 29, 33, 35, 39, 41, 45, 47, 49
3. Solve exercises 10.4: 1, 7, 10, 17, 20, 25, 26, 27, 29
5. Solve exercises 10.6: 1, 3, 5, 9, 12, 26, 27, 29
6. Additional “in-depth” problems:
7. The Diffusion Equation:

**Diffusion** is the net movement of a substance (e.g., ions or molecules) from a region of high concentration to a region of low concentration. This is also referred to as the movement of a substance down a concentration gradient. A gradient is the change in the value of a quantity (e.g., concentration, pressure, temperature) with the change in another variable (e.g., distance). For example, a change in concentration over a distance is called a concentration gradient. Diffusion is an important mechanism for transport in many biological systems.

The concentration of diffusing particles \( c(x, t) \) in one spatial dimension obeys the **partial differential equation**

\[
\frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial x^2},
\]

where \( x \) is the position in space, \( t \) is time, and \( D \) is the “diffusion constant”, which depends on the substance that is diffusing and the substrate in which it is diffusing.

(a) If \( c \) is in molecules/mm\(^3\), \( x \) is in mm, and \( t \) is in seconds, What are the units of \( D \)?

(b) Verify that the following function is a solution to the diffusion equation

\[
c(x, t) = \frac{1}{\sqrt{4\pi Dt}} \exp \left( -\frac{x^2}{4Dt} \right), \quad t > 0,
\]

(c) The solution above is a Gaussian function (bell shaped curve) for fixed \( t \). Describe what happens to the solution as time \( t \) increases.

8. Energy expenditure during locomotion:
The average energy $E$ (in kcal) needed for a lizard to walk or run a distance of 1 km has been modeled by the equation

$$E(m, v) = 2.65 \frac{m^{2/3}}{v} + \frac{3.5 \ m^{3/4}}{v}$$

where $m$ is the body mass of the lizard (in grams) and $v$ is its speed (in km/hr). Find the linearization of the energy function at $(m, v) = (400, 8)$ (400,8). (from C. Robbins, 1993).