CALCULUS, Math 17C
Homework 3 Due April 27

1. Read sections 10.6.

2. Solve exercises 10.6: 31,33,35,39,43,45,57,65,67.

3. Consider the function $f(x, y, z) = xy + xz + yz$.

   a) Compute the gradient vector of the function.

   b) What is the rate of change of f in the direction $v = (2, 1, -2)$ at the point $(1, 1, 2)$.

   c) What is the direction with the maximal rate of change at the point $(1, 1, 2)$.

4. In nature biological structures tend to take optimal shapes and forms. Plant cells are approximately box-shaped (i.e., rectangular-face solids). For a fixed volume V_0, what are the dimensions, length x, width y and height z of a cell, that minimizes the membrane surface area of the cell.

5. Experimental Science without understanding the data is useless speculation. Thus one often wishes to fit an equation (i.e., a model) to data. We want to find the values of the slope m and the vertical intercept b in the equation $y = mx + b$ that best fits a set of N data points (x_i, y_i) for $i = 1, \ldots, N$. For input x_j, the model predicts $\tilde{y}_j = mx_j + b$, so the error between a data point and the model prediction is

$$e_j = y_j - \tilde{y}_j = y_j - (mx_j + b).$$

   The *least-squares method*, invited by the prince of mathematics Carl Friederich Gauss around 1820, finds the values of m and b that minimize the sum of the squared errors between the data and the equation, i.e., m and b are the solution for the minimization of the (two-variable) problem:

$$E(m, b) = \sum_{j=1}^{N} (e_j)^2.$$

   a) Suppose we have three data points $(1, 0.5), (2, 1.1), (3, 1.4)$ find the values of m, b that minimize $E(m, b)$ in this case. Plot the data and the model equation $y = mx + b$ on the same graph to see what the least-squares fit looks like.

   b) Can you derive a general formula for the values of m, b that minimize $E(m, b)$ for an arbitrary set of N data points (x_i, y_i)?

Here is some *R*-code to fit a line to the data points using least-squares.

```
x<-c(1,2,3) y<-c(0.5,1.1,1.4)
plot(x,y)
mbfit=lm(y ~ x)
summary(mbfit)
abline(mbfit,col="red")
```