CALCULUS, Math 17C
Homework 4 Due May 11

1. Read sections 11.1,11.2
2. Solve exercise 11.1: 3,5,7,9,11,15,19,21.
3. Solve exercise 11.2: 3,7,9,11,19,23,27.
4. REST OF THIS HOMEWORK IS A REVIEW OF CHAPTER 10. COME READY ON WEDNESDAY MAY 3rd to discuss solutions in Class!!!

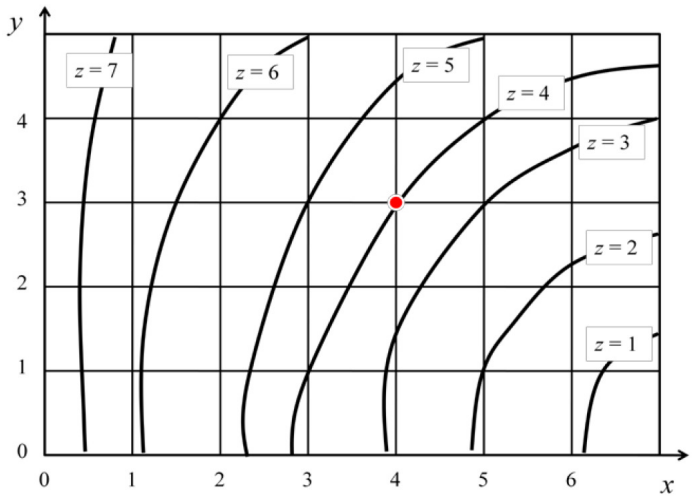
Problem 1. [20 points]

(a) [10 points] Consider the contour map of $z = f(x, y)$ below.

(i) Approximate $f(4,3)$.

(ii) Note that one can show that $\frac{\partial f}{\partial y}(4,3) \approx \frac{2}{3}$. Approximate the value of $\frac{\partial f}{\partial x}(4,3)$.

(iii) What is the approximate the rate of change of $f(x, y)$ at (4,3) in the direction $\vec{v} = (1,2)$?



PROBLEM 2: [20 points]

The Intelligence Quotient (IQ) in psychology is given by

$$Q = q(m, a) = 100\frac{m}{a}, \quad m > 0, a > 0,$$

where m is a person's "mental age" and a is their chronological (actual) age.

(a) [10 points] Determine and plot the contours of $Q = f(m, a)$ for:

(i) $Q = 100$ (average IQ),
(ii) $Q = 200$ (high IQ),
(iii) $Q = 50$ (low IQ)

(b) [10 points]

Suppose that, an individual is taking MAT17C and that (1) the individual's mental age (m) is 20 years old and is increasing at a rate of 2 mental years/year, and (2) their chronological age (a) is 20 years old (which always increases at and at a rate of 1 year/year). At this particular time, what is their IQ, and what is the instantaneous rate of change of the IQ with time?

That is, determine (i) the value of $Q = q(m, a)$ at (20,20), and (ii) the directional derivative of $Q = q(m, a)$ at (20,20) in the direction of $\vec{v} = (2,1)$.

(b) [10 points] A pharmaceutical company is developing a new antibiotic. Based on previous data, the company finds that the cost in hundreds of thousands of dollars to develop an antibiotic can be approximated by

$$C = f(x, y) = 5 + 2x^3 - 6xy + 3y^2$$

where *x* is the number of employees (in hundreds) working on quality assurance and *y* is the number of employees (in hundreds) working in the laboratory.

(i) Show that (1,1) is a critical point of $f(x, y)$, and use the second derivative test to verify that it is a local minimum.

(iii) What is the number of employees working on quality assurance and number of employees working in the laboratory that minimizes the cost to develop the antibiotic? What is this minimum cost?

PROBLEM 4 [10 points]:

(a) [5 points] If a function $f(x, y)$ has a local minimum at (1,1), then the directional derivative of $f(x, y)$ at (1,1) in any direction is strictly positive. True or False? Briefly justify your answer.

(b) [5 points] Consider the function $f(x, y) = x + y$ at the point $(x_0, y_0) = (0,0)$. How many directions are there in which the rate of change of $f(x, y)$ at (0,0) is exactly 2? Briefly justify your answer.