CALCULUS, Math 17C
Homework 7, Due June 8

1. Read sections 12.3, 12.4.

2. Solve exercise 12.3: 16,18,20,24,26,28,30,38,40.

3. The problems below are review for Midterm 2:

   For chapter 11 you will need to review/know:

   - systems of linear differential equations (11.1): how to write systems in matrix form, how to find and interpret the general solution eigenvalues and eigenvectors, and how to classify steady states (11.1.3).
   - compartmental models (11.2): know how to solve word problems (e.g. on drug administration) similar to those described in 11.2.1. (using method 11.1)
   - systems of nonlinear differential equations (11.3 and 11.4): know how to find steady states and determine their stability analytically (using the linearization/ Jacobian matrix).

   From chapter 12 you need to master:

   - 12.1, 12.2: COUNTING and BASIC IDEAS OF PROBABILITY. Be able to compute probabilities of several basic situations.
   - 12.3 CONDITIONAL PROBABILITY, INDEPENDENCE, BAYES THEOREM. E.g., Know how to solve drug test, diagnostic test problems.
   - Problems listed in next page will be discussed June 5th. Come prepared.

- The biochemical signaling molecule IP3 diffuses from one cell into another through intercellular gaps. Let $x_1(t), x_2(t)$ represent the concentration of IP3 in cell 1,2 respectively. If the dynamics is given by the system

$$\frac{dx_1}{dt} = -x_1 + x_2, \quad \frac{dx_2}{dt} = x_1 - x_2$$

  a) Use the differential equations to show the total amount of IP3 is constant in the cells. b) What is the steady state of the system? What is the general solution? c) Describe what happens when to x_1, x_2 when t goes to infinity? What is the meaning biologically? d) Suppose the initial condition is $x_1(0) = 2$ and $x_2(0) = 0$. What is the specific solution of the system?

- A system is described by

$$\frac{dx}{dt} = -x + 2m, \frac{dm}{dt} = \frac{2x}{x+1} - m$$

  .

  Find steady states and find the stability of the point. Classify!

- A DNA sequence is a succession of letters representing the primary structure of a DNA molecule or strand with the capacity to carry genetic information. The possible letters are A, C, G, and T representing the four nucleotide bases of a DNA strand - adenine, cytosine, guanine, thymine bases that are covalently linked to a phosphobackbone.

  Determine the total number of possible DNA strands that consist of exactly 11 bases, i.e., letters. (This is a measure of the maximal genetic information that is carried in the strand of DNA). But, how many distinguishable eleven-letter DNA sequences can be formed using the letters in the sequence AGTTGTTGCCG?

- A patient underwent a diagnostic test for hypothyroidism. The test correctly identifies patients who in fct have the disease 93% of the time and correctly identify healthy patients in 81% of the cases. If 4 out of 100 individuals have the disease, what is the probability that a test comes back negative?

- Bag contains 2 coins, one fair, one with 2 heads. You pick one of the coins with your eyes closed and flip it. What is the probability that you picked the fair coin, given that the outcome of the toss was heads?