Math 67
Homework 1 Due Friday January 12th

IMPORTANT: Please hand-in calculational exercises separated from proof-writing exercises (there will be two piles). BONUS exercises don't have to be hand-in. I separated the list of exercises in the three categories for your convenience.

1. Solve all calculational exercises in chapter 1 (page 9)
2. Solve calculational exercises 1,2,4,5(a,d), 6(a,d), 7. from chapter 2

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3. Solve all Proof-writing exercises in chapter 1 (page 10)
4. Solve proof-writing exercises 2, 3, and 5 from chapter 2.
5. In the planet ZZ the population has a different way to add two real numbers $a\overline{++}b = 2a + b$ (the + sign is on the right-hand side symbol is the earthlings way to do addition!!) Is this operation commutative or associative? why or why not?

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6. (BONUS) We defined in class addition and multiplication "modulo 6". Recall that this is done with the numbers $0, 1, 2, 3, 4, 5$ you add and multiply the usual way but then the final answer is the remainder of dividing by 6. For example $4 \cdot 5$ is normally equal 20 but modulo 6 (divide by 6) the answer has remainder 2, thus $4 \cdot 5 = 2$. Similarly $2 \cdot 3 = 0$ and $4 + 4 = 2$, $3 + 4 = 1$, etc. Do the numbers $0, 1, 2, 3, 4, 5$ with these operations form a field? How about modulo 7 addition and multiplication with the numbers $0, 1, 2, 3, 4, 5, 6$? Check all the properties of a field!
7. (BONUS): Can you think of a field with only 4 elements? Can you invent a field with 6 elements?