Math 67
Homework 1 Due January 23th

IMPORTANT: Please hand-in calculational exercises separated from proof-writing exercises (there will be two piles). BONUS exercises (if any) don't have to be handed-in. I separated the list of exercises in the three categories for your convenience.

1. Solve all calculational exercises 1(a), 2 in chapter 3
2. Solve calculational exercises 1,2,3(a,c,f),4(b,d,i,j) 5(c,h,i) 6(c,d), 7(a,c), 8(a,h,q). from chapter 12.
3. Solve the following system of linear equations using the RREF Gaussian elimination method.

$$\begin{array}{rrrrrl} x_1 & +x_2 & +x_3 & -2x_4 & -2x_5 & =5 \\ 2x_1 & -x_2 & & -x_4 & -2x_5 & =0 \\ 2x_1 & & -x_3 & -2x_4 & -x_5 & =0 \\ & x_2 & -x_3 & -x_4 & +x_5 & =0 \end{array}$$

$$\begin{array}{rrrrl} x_1 & -3x_2 & +4x_3 & +6x_4 & =0 \\ -2x_1 & +4x_2 & +x_3 & +7x_4 & =0 \\ 3x_1 & -x_2 & +2x_3 & +5x_4 & =0 \\ -x_1 & +2x_2 & +3x_3 & +7x_4 & =0 \end{array}$$

4. For each of the following matrices find a RREF matrix equivalent to it and write the associated factorization via a product of elementary matrices:

$$A_1 = \begin{bmatrix} 1 & 2 & 1 & 0 \\ -1 & 0 & 3 & 5 \\ 1 & -2 & 1 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 2 & 0 & i \\ 1 & -3 & -i \\ i & 1 & 1 \end{bmatrix}$$

5. Find all solutions of the system of equations:

$$(1-i)x_1 - ix_2 = 0$$

$$2x_1 + (1-i)x_2 = 0$$

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6. Solve proof-writing exercises 1,3 in chapter 3

7. Solve proof-writing exercises 1,3,4 from chapter 12.

8. Find two examples of 2×2 matrices with the property that $A^2 = 0$ but $A \neq 0$.

9. For which values of k does the following system have a solution:

$$\begin{array}{lllll} x_1 & +x_2 & +x_3 & +x_4 & = 1 \\ x_1 & & +x_3 & & = 1 \\ & x_2 & & +x_4 & = k \end{array}$$

10. for which values of (a, b, c) does the following system have a solution?

$$\begin{array}{rrrl} 3x_1 & -x_2 & +2x_3 & = a \\ 2x_1 & +x_2 & +x_3 & = b \\ x_1 & -3x_2 & & = c \end{array}$$

11. Suppose A is a 2×1 matrix and B is a 1×2 matrix. Show that AB is not invertible.

12. Recall that a square matrix is upper triangular if $A_{ij} = 0$ for $i > j$. Show that a square upper triangular matrix is invertible if and only if each of its diagonal elements are non-zero.