Math 67
Homework 4 Due February 13th

IMPORTANT: Please hand-in calculational exercises separated from proof-writing exercises (there will be two piles). BONUS exercises (if any) don't have to be handed-in. I separated the list of exercises in the three categories for your convenience.

1. Solve calculational exercises 2, 3(b,d), 4, 5 in chapter 5.

2. Let $B = \{(1, 0, i), (1 + i, 1, -1)\}$ and call W the subspace of $\mathbb{C}^3$ spanned by B. Show that B is a basis for W. Is $B' = \{(1, 1, 0), (1, i, i + 1)\}$ a basis for W?

3. Consider W, the row space of the 3×4 matrix.
$$\begin{bmatrix} 1 & 2 & 2 & 1 \\ 0 & 2 & 0 & 1 \\ -2 & 0 & -4 & 3 \end{bmatrix}$$
Recall this is a subspace of R^4.
a) Show that its rows form a basis for W
b) If (b_1, b_2, b_3, b_4) is a vector of W, what are the coordinates of it in terms of the basis of part (a)?
c) Show that $(1, 0, 2, 0)$, $(0, 2, 0, 1)$, and $(0, 0, 0, 3)$ is another basis for W.
d) What is the dimension of the column space of the same matrix?

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4. Solve proof-writing exercises 1,2,4,5,6,7 in chapter 5.

5. Let $R^{m \times n}$ be the vector space of matrices with coefficients over the reals. Is this space finite-dimensional? If yes, what is its dimension?

6. Let V be the vector space $R^{2 \times 2}$ as above. Let W_1 be the set of matrices of the form
$$\begin{bmatrix} x & -x \\ y & z \end{bmatrix}$$
and let W_2 be the set of all matrices of the form
$$\begin{bmatrix} a & b \\ -a & c \end{bmatrix}$$
a) Show that W_1,W_2 are subspaces of V.
b) Find the dimension of the subspaces $W_1, W_2, W_1 + W_2$, and $W_1 \cap W_2$.

7. Is every subspace of a finite-dimensional vector space finite-dimensional?

8. If V,W are vector spaces of dimensions n, m respectively what is the dimension of $V \bigoplus W$?

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9. (BONUS) Can a proper subspace of a finite-dimensional vector space have the same dimension as the whole space?

10. (BONUS) Unlike the reals or the complex, there are fields with finitely many elements. If a field F has q many elements, you can still construct F^3. How many elements does it have? how many different bases are there?