Math 67
Homework 5 Due February 20th

IMPORTANT: Please hand-in calculational exercises separated from proof-writing exercises (there will be two piles). BONUS exercises (if any) don't have to be handed-in. I separated the list of exercises in the three categories for your convenience.

1. Solve calculational exercises 1,3,4,5,6 in chapter 6.
2. Consider the transformation from the vector space $R_6[x]$ to $R_7[x]$ given by

$$T(p(x)) = \int_{-3}^{x+9} p(t)dt$$

   Is T a linear transformation? if yes, what is the nullspace of T? What is its range?

3. Let T be the linear map from R^3 into itself defined by $T(x_1, x_2, x_3) = (3x_1, x_1 - x_2, 2x_1 + x_2 + x_3)$.

   a) Is T an invertible map? If yes, find an expression for T^{-1} otherwise show evidence that it is not.

   b) Prove that $(T^2 - I)(T - 3I) = 0$.

4. Give a description of a linear map from R^3 into R^3 whose range is the subspace generated by the vectors $(1, 0, -1)$ and $(1, 2, 2)$
5. Let T be the linear operator in R^3 defined by $T(x_1, x_2, x_3) = (3x_1 + x_3, -2x_1 + x_2, -x_1 + 2x_2 + 4x_3)$.

   a) Find the matrix associated to T using the canonical basis for R^3.

   b) Then find the matrix associated with the basis $(1, 0, 1)$, $(-1, 2, 1)$, and $(2, 1, 1)$.

   c) Prove that T is invertible and give an expression for T^{-1} similar to the expression we used to define T above

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6. Solve proof-writing exercises 1, 2, 7, 9 in chapter 6
7. Let V be a vector space of dimension n. Let T be a linear operator on V with the property that the range and the nullspace of T are the same. Show that n has to be an even number. Can you construct an example of such operator explicitly?
8. Let T be a linear operator over a finite-dimensional vector space V. Suppose that $dim(range(T)) = dim(range(T^2))$. Show that the range and the nullspace of T intersect only at the zero vector.

9. For a field F, is $F^{m \times n}$ isomorphic to F^{mn}?

10. If V is a finite dimensional vector space is $L(V, V)$ finite-dimensional?

11. What happens to the matrix of a linear operator on a finite-dimensional vector space when the elements of the basis used are permuted among themselves?

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12. (BONUS) A matrix is 0/1 if all its entries are either 0 or 1. What is the largest number of 1's that an invertible ($n \times n$) 0/1 matrix can have?

13. (BONUS) Which $n \times n$ matrices B have the property that $AB = BA$ for **all** $n \times n$ matrices A?