Math 67
Homework 6 Due March 2nd

IMPORTANT 1: The second midterm exam is Monday March 2nd. Some tips:

1) Make sure you read and understand Chapters 5-7 in the book. These are the topics that we will cover in the exam.

IMPORTANT: Chapter 8 **will not** appear in the exam, and I moved those exercises to Homework 7.

2) Learn how to use the key theorems/formulas in those chapters. Memorize the key definitions and terminology.

3) There will be 5 questions in the exam.

IMPORTANT 2: Please hand-in calculational exercises separated from proof-writing exercises (there will be two piles). PRACTICE exercises don't have to be handed-in. I separated the list of exercises in the three categories for your convenience.

——————CALCULATION——————

1. Solve all calculational exercises 1,2,3,4,5,7 in Chapter 7.
2. Say as much as you can about the eigenvalues and eigenvectors of the linear transformation defined on C^3 by $T(x_1, x_2, x_3) = (x_2, x_3, x_1)$.
3. Find the eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 9 & 4 \\ 1 & 6 \end{bmatrix}$
4. Suppose A is a 3×3 symmetric matrix with eigenvalues 2,5,7 and corresponding eigenvectors x_1, x_2, x_3. Suppose $x = 4x_1 - 5x_2 + x_3$ Find Ax and $x^T Ax$.

——————PROOF-WRITING——————

5. Solve proof-writing exercises 1,2,3,4,8,11 in Chapter 7.
6. If T is a linear transformation on a finite-dimensional space and $p(x)$ is a polynomial, what information do the eigenvalues of T give to $p(T)$?
7. Let V the vector space of polynomials with coefficients over R. Let T be the linear operator defined by the integral

$$T(f)(x) = \int_0^x f(t)dt$$

Show the T does not have eigenvalues.

——————-PRACTICE TEST——————

8. (PRACTICE) Find a matrix for the linear transformation $T : R^4 \to R^2$ given by $T(x, y, z, w) = (x - y, x + y + 2w)$. Find a basis for the range and for the nullspace of T. Is the map T invertible?

9. (PRACTICE) Find a basis for the vector space of all real 3×3 symmetric matrices.

10. (PRACTICE) Determine the dimension of the subspace

$$\{(x_1, x_2, x_3, x_4) \in F^4 | x_4 = x_1 + x_2, x3 = x_1 - x_2\}$$

11. (PRACTICE) Let A be a 4×3 matrix whose columns are linearly independent. What is the dimension of the column space? what is the dimension of the Nullspace of the linear transformation associated to A?

12. (PRACTICE) Suppose you have two injective linear maps $S : U \to V$ and $T : V \to W$ show that the composition TS is injective.

13. (PRACTICE) Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 2 & 2 \\ 3 & 2 & 1 \end{bmatrix}$. Answer the following questions

    a) Is A invertible?

    b) Is $A + 2I$ invertible?

    c) Find all eigenvalues and eigenvectors of A.

14. (PRACTICE) Suppose V is a finite-dimensional vector space over F and U is a subspace. Show that if $dim(V) = dim(U)$ then $U = V$.