THEORY OF NUMBERS, Math 115 A
Homework 2 Due Wednesday October 9

1. Use the sieve of Eratosthenes to find all primes less than 250
2. Find all primes that are difference of the fourth powers of two integers.
3. Show that there are no "prime triplets", that is, primes $p, p+2, p+4$, other than 3,5,7.
4. Let $Q_n = p_1 p_2 ... p_n + 1$ where $p_1, \ldots, p_n$ are the n smallest primes. Determine the smallest prime factor of Q_n for $n \leq 6$.
5. Prove that no polynomial function with integer coefficients can take only prime values when evaluated at the integers. HINT: Use the binomial theorem (i.e the expansion of $(x+y)^p$ in terms of binomial coefficients) and the fact that a polynomial of degree d cannot repeat too many times the same value.
6. A prime power is an integer of the form p^k, where p is prime and n is a positive integer greater than 1. Find all prime powers that differ by 1 and prove that your answer is correct.
7. Find the greatest common divisor of each of the following sets of integers: (a) 100, 121, (b) 1001, 289 (c) 6,15,21. For the two items please write the gcd as a linear combination of the numbers.
8. Show that the gcd of $8a+3$ and $5a+2$ is always one.
9. Every positive integer greater than 6 is the sum of two relatively prime integers greater than 1.