THEORY OF NUMBERS, Math 115 A
Homework 3 Due Wednesday October 16

1. Use the Euclidean and extended Euclidean algorithms to find the gcd of the following pairs of numbers and to represent them as a linear combination of the input data:

   a) $(45, 75)$, b) $(666, 1414)$, c) $(102, 222)$ d) $(20785, 44350)$.

2. Let m, n be positive integers and let a be an integer greater than 1. Show that $(a^m - 1, a^n - 1) = a^{(m,n)} - 1$.

3. Show that every positive integer can be written as the product of possibly a square and a square-free integer. Recall a *square free* integer is an integer that is not divisible by any perfect square other than 1.

4. Let $N(a+b\sqrt{-5}) = a^2+5b^2$. Show that if $m = a+b\sqrt{-5}$ and $n = c+d\sqrt{-5}$, where a, b, c, d are integer, then $N(nm) = N(n)N(m)$.

5. Show that the numbers $1 + \sqrt{-5}$ and $1 - \sqrt{-5}$ are prime numbers inside the set of special numbers $a + b\sqrt{-5}$. Hint: use the previous exercise.

6. Find the least common multiple of each of the following pairs of integers:

   a)7,11 b) 101,303 c) 1331, 5005 d) 5040, 7700.

7. Show that $\sqrt{2} + \sqrt{3}$ is irrational.

8. Prove that there are infinitely many primes of the form $6k + 5$, where k is a positive integer.