THEORY OF NUMBERS, Math 115 A
Homework 6 Due Wednesday November 15 2002

1. (6.1 6) What is the remainder when $7 \cdot 8 \cdot 9 \cdot 15 \cdot 16 \cdot 17 \cdot 23 \cdot 24 \cdot 25 \cdot 43$ is divided by 11?

2. (6.1 12) Using Fermat's little theorem, find the least positive residue of $2^{1000000}$ modulo 17.

3. (6.1 34) Show that if p is a prime and $0 < k < p$, then $(p-k)!(k-1)! \equiv (-1)^k$ modulo p.

4. (6.1 40,41) Using the fact that p divides the binomial coefficient $\binom{p}{k}$ when k is less than p show that if a, b are integers then $(a+b)^p \equiv a^p + b^p$. Use this to prove Fermat's little theorem.

5. (6.2 2) Show that 45 is a pseudoprime to the bases 17 and 19.

6. (6.2 20) Show that if n is a Carmichael number then n is square free.

7. (6.2 1 computer exploration) Determine for which positive integers n, $n \leq 100$, the integer $n2^n - 1$ is prime.

8. (6.3 6) Find the last digit of the decimal expansion of $7^{999,999}$.

9. (6.3 10) Show that $a^{\phi(b)} + b^{\phi(a)} \equiv 1 \ (mod\ ab)$ if a,b are relatively prime positive integers.

10. (6.3 2 computational exploration) Find $\phi(n)$ for all integers less than 1000. What conjecture can you make about the values of $\phi(n)$.