THEORY OF NUMBERS, Math 115 A
Homework 7 Due Wednesday November 27 2002

1. (not from the book) Prove or disprove the following statement: For every odd number n, $\sigma(n) \leq 2n - 1$. HINT: Try to experiment with LOTS of numbers, use your intuition.
2. (7.1, 8) Show that there is no positive integer n such that $\phi(n) = 14$.
3. (7.1, 18) Show that if m, k are positive integers then $\phi(m^k) = m^{k-1}\phi(m)$.
4. (7.2, 12) Show that the equation $\sigma(n) = k$ has at most a finite number of solutions when k is a positive integer.
5. (7.2, 11) What is the product of the positive divisors of a positive integer? Write a formula for it.
6. (7.3, 4) Find a factor of each of the following integers: $2^1 11 - 1$, $2^{289} - 1$, and $2^{46189} - 1$.
7. (7.4, 6) Find all composite numbers n between 100 and 200 with $\mu(n) = -1$.
8. (7.4, 8) Find the value of the Mertens function $M(n)$ for $n = 100$.
9. (7.4, 22) Let n be a positive integer. Show that $\Pi_{d|n}\mu(d)$ equals (1) -1 if n is a prime, (2) Zero if n has a square factor, and (3) 1 if n is square-free and composite.