THEORY OF NUMBERS, Math 115 B

Homework

1. Prove that any triangulation of a simple polygon with $n$ vertices has $n - 3$ diagonals and $n - 2$ triangles.

2. Prove that any triangulation of a simple polygon has an ear: i.e. a triangle that shares a single edge.

3. Suppose $a, b$ are relatively prime integers. Prove that there are no lattice points in the interior of the segment going from $(0, 0)$ to $(a, b)$.

4. Determine the number of lattice points in the segments $(12, 24) - (17, 20)$ $(3/4, 8/7) - (3, 11/7)$.

5. Find out the number of lattice points inside the polygon $P = v_0, v_1, ..., v_5$ with vertices $v_0 = (0, 0), v_1 = (1, 1), v_2 = (2, 4), v_3 = (3, 9), v_4 = (4, 16), v_5 = (5, 25)$.

6. Using straight line segments, draw a nonconvex figure with symmetry about $(0, 0)$, area more than 4, and no lattice points except $(0, 0)$ inside it or on its boundary.

7. Find a tetrahedron with integer coordinates for its vertices but no lattice points on its boundary or interior, whose volume is more than $1/6$. This makes a generalization of Pick’s theorem impossible.

8. A convex set $Q$ contains three non-collinear points $A, B, C$. Prove that then $Q$ must contain every point of the triangle $ABC$.

9. (application of Minkowski theorem) Using Minkowski’s geometric thinking prove the following number theory statement: Given any real number $\alpha$ and an integer $t > 0$, there exist integers $p, q$ no both zero such that $|q/p - \alpha| \leq 1/t$. (HINT: take the right M-set and apply Minkowski’s theorem to it).